

# Bayesian Persuasion in the Presence of Discretionary Disclosure

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## Abstract

We consider a firm's design of its financial reporting system when the firm incurs a significant loss if it does not meet a crucial threshold. If the financial reports were the only basis for updating beliefs by the outside party responsible for applying the threshold, the firm's problem would fall into a class of sender-receiver games referred to as Bayesian persuasion. A key result in such games is a preference by the sender for imperfectly informative reports that enhance the likelihood of meeting, but not necessarily exceeding the threshold. Our innovation with respect to this class of games is the addition of a later stage at which the firm may receive private information and, if so, has discretion over its disclosure. This additional stage may induce the choice of a more or less informative financial reporting system than would otherwise be chosen, depending on the properties of the private signal and the prior beliefs. The net result of such a choice is therefore to change the probability of meeting the threshold. We show that this choice always reduces the firm's expected benefit. Interestingly, an equilibrium in which the firm never discloses its private information may also exist. Setting feasibility aside, mandatory disclosure of private information may or may not increase the firm's expected utility.

# 1 Introduction

Financial reports issued by firms are often used by outside parties to form beliefs and make decisions. In many cases, the outside parties employ decision rules based on whether their expectations based upon those beliefs meet certain thresholds. If the outsiders' expectations fall short of the required threshold, the firm incurs losses. In this paper we use a setting that involves such a threshold decision rule to explore the interaction between the selection of financial reporting policies and the discretionary disclosure of private information. Common examples of settings involving threshold decision rules include external audits, credit ratings, covenant requirements, regulated bank capital ratios, major impairment tests, and business certifications. Firms that fail to meet these types of thresholds often incur losses associated with qualified audit opinions, speculative bond ratings, poor credit rankings, covenant violations, low capital ratios, asset impairments, or sub-standard business classifications. Although financial reports serve many other purposes, it is not difficult to envision situations where the incentive to meet some prescribed threshold is sufficiently important to warrant consideration in the design of systems generating those reports. We exploit the simplicity of a threshold decision rule setting to examine, in a parsimonious model, how the *ex ante* design of a firm's financial reporting system can be affected by the potential for a firm to later receive reportable private information.

Models examining the design of information systems, where the sender can commit to a design that is observable to the intended receivers, are referred to as Bayesian persuasion models by Kamenica and Gentzkow (2011). These types of models are relatively new in the literature and have been studied, for example, in Goex and Wagenhoffer (2009), Duggan and Martinelli (2011), Gentzkow and Kamenica (2013), Michaeli (2014), Taneva (2014), Alonso and Camara (2014), and Wang (2013). Financial reporting systems fit the Bayesian persuasion framework reasonably well in the sense that firms have flexibility in choosing accounting policies that may advance their interests. We consider a setting in which the firm's overriding interest is to meet a crucial threshold. In our context, beyond the information supplied by financial reports, firms are likely to receive private information the disclosure of which is discretionary. Press releases, public announcements, management forecasts, and supplemental SEC filings are among the more familiar conduits through which private information may be disclosed. Disclosure or non-disclosure of this information may also contribute to the formation of posterior expectations affecting whether thresholds are met. As such, anticipation of receiving private information may

factor into the design of financial reporting systems. Our study explores this interaction. We also compare discretionary and mandatory disclosure of private information in assessing the value to the firm of the option to not disclose. Stepping back, we consider the value of private information *per se*, i.e., whether the firm is better or worse off when it may become privately informed.

In a pure Bayesian persuasion context, firms facing threshold concerns may seek to dampen the informativeness of financial reports. The basic idea is that the firm may increase the relative frequency of reports that are just good enough to meet the threshold by allowing imperfection in reporting of states that would exceed the threshold. For instance, suppose that a perfectly informative, but relatively infrequent, good report implied a state that strictly exceeded the threshold. By allowing a good report to also sometimes be generated in a state that (if perfectly observed) would not meet the threshold, the firm may be able to increase the relative frequency of a good report, thereby still meeting the threshold but with higher *ex ante* probability. In a vernacular familiar to accountants, threshold concerns create an incentive for introducing a liberal bias into financial reporting systems. Biases motivated in response to threshold concerns can therefore manifest in liberal accounting policy choices.

Many studies in accounting employ models of financial reporting systems with state-dependent asymmetric informativeness, or bias, similar to our model. Gigler and Hemmer (2001) show how a conservative bias may reduce pre-emptive voluntary disclosure, thereby mitigating the value of communication between managers and shareholders. While they seek to address the question of how reporting quality affects discretionary disclosure, we seek to address how the availability of a discretionary disclosure channel influences properties of public reporting systems. Kwon, Newman, and Suh (2001) consider optimal compensation arrangements in a moral hazard context with limited liability for which bad reports are less informative and good reports more informative of underlying bad and good states, respectively. Gigler, Kanodia, Sapiro, and Venugopalan (2009) show how bias in a reporting system may make it more or less likely that a favorable or unfavorable signal accurately reports the underlying state in a setting where investment continuation decisions are at stake. Beyer (2012) considers an aggregate reporting system for a multi-segment firm that only reports losses and not gains in asset values. Such a system is less informative about gains in values, but is more informative about losses by comparison with a system that reports both since losses are not offset by gains.

Friedman, Hughes, and Saouma (2015) portray effects of reporting biases on competition. Of particular interest is the distinction they draw between bias and precision in showing how bias may increase overall reporting system informativeness holding symmetric precision constant.

The addition of a subsequent stage at which firms may or may not disclose private information not encompassed by its financial reports influences the optimal design of financial reporting systems in a surprising way. We identify conditions on the informativeness of private signals and on the prior beliefs such that, in anticipation of the effects of discretionary disclosure on posterior expectations, firms choose more informative financial reporting systems that reduce the probability of meeting the thresholds in comparison to the case where firms do not expect to receive private information.<sup>1</sup> As we elaborate below, a design that provides more informative financial reports of states that exceed the threshold may be necessary to overcome the reduction in posterior expectations from non-disclosure of a private signal. Increasing the informativeness of the reports comes at a cost: a reduced frequency of reports that cause beliefs to meet the threshold. We further find that the option to not disclose may or may not be valuable to the firm in comparison to mandatory disclosure (presuming that such mandatory disclosure could be enforced). Stepping back to consider the impact of private information on the firm's welfare, we find that the firm may be better off in meeting a crucial threshold without the potential to receive private information. Last, we consider the efficacy of commitments not to disclose private information and find that in the absence of some benefit to disclosure beyond meeting the threshold, such commitments are sustainable.

As is typical in models of discretionary disclosure (e.g., Verrecchia 1983, Dye 1985 and Jung and Kwon 1988), in equilibrium, a low-end pool is formed and private information that would lower posterior expectations is suppressed. Only signals that would raise posterior expectations above the prior expectations are disclosed. Of course, rational receivers would lower their expectations upon not observing a disclosure to take into account that the sender may have realized a low signal. Holding constant the design of the financial reporting system with no private information, raising expectations from those induced by financial reports may be excessive to maintaining the highest probability of just meeting the threshold. While the firm may adjust for this effect by making the financial reporting system less informative, this

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<sup>1</sup> Also possible is the somewhat more intuitive case for discretionary disclosure of private information to induce firms to choose less informative financial reporting systems to offset the anticipated effects of information contained in such disclosures.

response is mitigated by the need to offset the effect of non-disclosure in lowering expectations. Hence, as we alluded to in summarizing our results, interesting questions are whether the option to not disclose private information is valuable to the firm and whether either discretionary or mandatory disclosure of private information is valuable to the firm in meeting a crucial posterior expectations threshold.

Among the issues we have suppressed in our model is the prospect of *ex post* manipulation of financial reports. In the absence of some added friction or noise, we can ignore such biasing since rational receivers of those reports will undo their effects. We could also allow for biases that cannot be undone as long as there are limitations on a firm's flexibility in distorting reports before they are disseminated. The important feature of the financial reporting system structure in our model is that one cannot completely undo the effects of *ex ante* design choices *ex post*. We also ignore any out-of-pocket costs to increasing the informativeness of the financial reporting system; a perfectly informative system is feasible at no such cost. Introducing an out-of-pocket cost of a more informative reporting is unnecessary as a friction to prevent a corner solution and would merely obscure the insight that a less than perfectly informative system may be desirable as a means of inducing beliefs that meet a threshold with greater probability.

There is considerable empirical support regarding the importance of meeting thresholds in avoiding losses. Kausar, Taffler, and Tan (2006) find that institutional investors tend to divest after going concern qualifications. Menon and Williams (2010) find negative market reactions to going concern qualifications in audit reports likely driven by the dependencies of exchange listings, debt terms, and financing on obtaining unqualified reports. Graham and Harvey (2001) find that credit ratings are a major concern for CFOs in capital structure decisions, while Kisgen (2006) notes that an inability to maintain high ratings may exclude institutions from holding bonds, trigger higher interest rates, etc., thereby affecting capital structure decisions. Beneish and Press (1993) find that violations of debt covenants lead to increases in interest rates, and in a later study Beneish and Press (1995) detect negative market reactions associated with such violations. Li, Shroff, Venkataraman, and Zhang (2011) find indirect evidence that failing to pass goodwill impairment tests was a principal concern of firms given the negative impact of impairments on analysts' and market expectations.

As well, there is empirical support for recourse to accounting practices as a device for

boosting the likelihood that thresholds would be met. Bartov, Gul, and Tsui (2001) find an association between discretionary accruals and audit report qualifications.<sup>2</sup> Press and Weintrop (1990) find that firms use accounting flexibility to meet debt covenants. Healy and Palepu (1990) find the opposite; however, Begley (1990) suggests that this could be an identification issue. Sweeney (1994) finds that firms approaching covenant violation early-adopt mandatory income-increasing changes and that firm's discretionary changes are increasing in default costs. Dichev and Skinner (2002) find that a large number of firms meet or beat covenants suggesting manipulation of reports upon which covenants are based. Kim and Kross (1998) find evidence of manipulation of loan loss provisions coincident with a change in bank capital standards. Ramanna and Watts (2012) find firms tend to use discretion in applying tests of goodwill impairment. Chen, Lethmathe and Soderstrom (2015), study the firm's reporting behavior when their objective is to meet a return level required to be accepted into a UN carbon emission program. Bonachi, Mara and Shalev (2015) find evidence consistent with parent firms accounting for business combinations under a common control at fair value when their leverage is high and they have net covenants.

Our study makes several contributions to the accounting literature. To the best of our knowledge, we are the first to model the impact of *ex post* discretionary disclosure of private information on the *ex ante* design of public reporting systems. Notwithstanding a high level of abstraction, our model captures an incentive for biased financial reporting distinct from other incentives characterized in the literature. Especially noteworthy is our result that a more informative financial reporting system induced by discretionary disclosure of private information may weaken the effect of the reporting system in raising the probability of meeting a crucial threshold.<sup>3</sup> The results we obtain on the value of discretionary disclosure of private information or the value of private information under either discretionary or mandatory disclosure offer further insights on the influence of meeting crucial thresholds that firms may face.

The most closely related paper to ours is Stocken and Verrecchia (2004). In their model, an *ex ante* choice of financial reporting system precision is followed by the sender's

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<sup>2</sup> Signed accruals have been a common workhorse for detecting earnings manipulation. We suggest that biased accruals could be an artifact of accounting policies chosen *ex ante* as well as a consequence of *ex post* manipulations. The former would appear to be more likely in a context where thresholds apply over multiple reporting periods.

<sup>3</sup> The opposite case of discretionary disclosure of private information inducing a less informative financial reporting system is also possible.

manipulation of a report based on the realization of the signal generated by that system and a further private signal realization. The sender's ability to manipulate the report *ex post* may induce a less precise *ex ante* reporting system choice. In contrast, our paper focuses on *ex ante* choices that affect precision and bias and the sender's ability to exercise discretion over disclosure of a private signal. In our paper the potential for discretionary disclosure can have a negative or *positive* effect on the informativeness of the reporting system chosen *ex ante*.

Another closely related study is Kamenica and Gentzkow (2011). They consider how an optimal information system will be set when the sender (the firm in our case) is uncertain about the beliefs of a receiver (the outside party in our case). In our model, given that the firm does not know if and what private information it will observe, there is also uncertainty about the beliefs of the outside party at the stage in which the reporting system is set. However, in our model, the firm has a partial control, because it can choose to disclose or withhold this private information. In this context, we show that the firm cannot, by discretionary disclosure of subsequently acquired private information, improve the likelihood of meeting the threshold beyond that achievable from the public reporting system alone; i.e., given a choice, the firm strictly prefers not to obtain private information.

## 2 The Model

There is a stochastic state affecting the payoff for a risk-neutral firm. The firm, through the design of a financial reporting system, seeks to maximize the probability that an outside party's posterior expectation of the state (i.e., after receiving reports and messages from the firm) at least meets a predetermined threshold. The firm experiences a significant benefit whenever the outsider's beliefs meet or exceed the threshold. Effectively, the outside party conferring the benefit based on the threshold decision rule is a passive Bayesian player in the game. As mentioned earlier, common examples attesting to the importance of meeting thresholds include avoiding a going concern qualification to an audit report, avoiding a downgrading of debt issues, avoiding covenant violations, avoiding sub-standard bank capital ratios, and avoiding asset write-downs in impairment tests.

The players have common prior beliefs. The accounting policies that comprise the firm's *ex ante* choice of its financial reporting system are publicly observable. We assume that the firm receives a private signal with a probability strictly less than one as in Dye (1985) and Jung and



Kwon (1988) after the financial reporting system has been implemented. This probability and the distribution generating private signals are also common knowledge. Disclosed signals are credible and it is not possible to credibly communicate not having received a signal.

For analytic tractability, we adopt a binary state and reporting structure similar to Gigler and Hemmer (2001), Kwon et al. (2001), Bagnoli and Watts (2005), Smith (2007), Chen and Jorgensen (2012), Guo (2012), and Friedman et al. (2015); albeit in a different context. While parsimonious, the structure is adequate for depicting persuasive behavior on the part of the firm in choosing its reporting system. A similar binary structure for the firm's private signal, if received, is sufficient for depicting the impact of discretion over disclosure on the reporting system design choice. In order to focus on the design of the financial reporting system, we assume a parameterization that preserves pooling of a low signal realization with non-receipt of a signal as a rational strategy.

Formally, the firm's random state is represented by  $\theta \in \{H, L\}$  where  $H$  and  $L$  represent high and low values, respectively. We normalize values by setting  $H = 1$  and  $L = 0$ . The outsider's threshold against which he compares posterior expectations is represented by  $k \in (0,1)$ . Common prior beliefs are defined by  $\alpha = \Pr(\theta = H)$ .<sup>4</sup> We assume  $\alpha < k$  to avoid the trivial case where the threshold is met even in the absence of additional information provided through reports and messages. The financial reporting system generates a report with the structure:

$$\Pr(r = g|\theta = H) = \beta_H \in [0,1]$$

$$\Pr(r = g|\theta = L) = \beta_L \in [0,1]$$

where  $\beta_H \geq \beta_L$ . The manager chooses  $\beta \equiv (\beta_H, \beta_L)$  prior to potentially receiving a signal  $s$ . With probability  $q \in (0,1)$ , the firm receives a non-empty private signal  $s \in \{h, l\}$  with the following structure:

$$\Pr(s = h|\theta = H, s \neq \emptyset) = \gamma_H \in [0,1]$$

$$\Pr(s = h|\theta = L, s \neq \emptyset) = \gamma_L \in [0,1]$$

where  $\gamma_H > \gamma_L$ . The firm cannot credibly communicate not having received a signal,  $s = \emptyset$ , which happens with probability  $1 - q$ . Upon receiving a non-empty signal  $s$ , the firm can either truthfully disclose that signal by sending a message  $m = s$  or not disclose, in which case the

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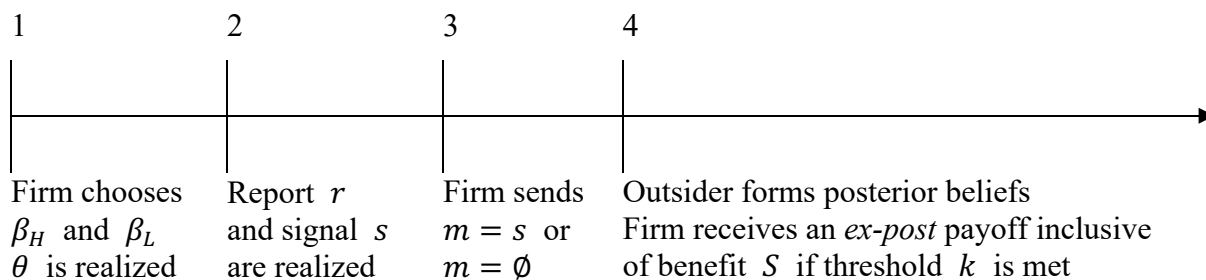
<sup>4</sup> In our model, beliefs are equivalent to the probability that the state is high, which, given our assumption that  $H = 1$  is also the expected value. As such, we tend to use expectation and beliefs interchangeably.

message  $m = \emptyset$  is the same as when a signal is not received.

We assume that the firm's payoff is increasing in the posterior expectation of the outside party about the firm's state. Of principal interest, the firm receives an additional benefit if the expected state meets or exceeds a threshold. Formally, we define the firm's *ex-post* payoff as:

$$\pi \equiv \sigma E[\theta|m, r] + 1_{E[\theta|m, r] \geq k} S$$

where  $S$  is the discrete benefit (or loss avoided) from meeting the threshold,  $k$ , normalized to 1, and  $\sigma > 0$  is the sensitivity of the *ex post* payoff to an increase in the posterior expectation.<sup>5</sup>



**Figure 1**  
Timeline of events

Figure 1 depicts the timeline of events. At date 1, the firm chooses the parameters  $\beta \equiv \{\beta_H, \beta_L\}$  governing the financial reporting system. The state,  $\theta$ , is drawn by nature, but observed by neither the firm nor the outsider. At date 2, the financial report is realized and observed by both players, and either a private signal  $s$  is realized and privately observed only by the firm, or no signal is received. At date 3, the firm sends either a message  $m = s$ , or  $m = \emptyset$  to the outside party. At date 4, the outside party forms a posterior expectation of the state and assesses whether the threshold has been met. The firm receives  $\sigma E[\theta|m, r]$  and an additional benefit  $S$  normalized to 1 if the outsider's posterior expectations meet or beat the threshold.

### 3 Analysis

#### 3.1 Optimization Objective

Using the Law of Iterated Expectations, the *ex ante* expected payoff of the firm simplifies to

<sup>5</sup> We discuss the case  $\sigma = 0$  later in Section 4.2.

$$E[\pi] = \sigma\alpha + B,$$

where

$$B \equiv E[1_{E[\theta|m,r] \geq k}]S = Pr(E[\theta|m,r] \geq k) \cdot 1 = Pr(E[\theta|m,r] \geq k)$$

is the expected benefit to the firm of meeting (or exceeding) the threshold which, under our maintained assumptions, is equal to the probability that the threshold is met. It is straightforward to show that maximizing the expected payoff of the firm is equivalent to maximizing the expected benefit  $B$ :

$$\operatorname{argmax}_{\beta \in X} E[\pi] = \operatorname{argmax}_{\beta \in X} B$$

where  $X$  is the set of plausible values of  $\beta$ . When choosing across regimes and reporting systems we compare only the expected benefit  $B$ . Because the firm's *ex ante* expected payoff apart from meeting the threshold is unaffected by its reporting system and message choices, the only way it can affect that payoff is by increasing the outside party's expected posterior probability sufficiently to meet the threshold, thereby receiving  $S$ . In other words, the only "production" lies in meeting the threshold.

### 3.2 Financial Reporting With No Private Information

We now consider a special case in which the firm never receives private information. This case is a pure persuasion game in which posterior expectations are based only on the firm's financial report. Consider the extreme choices of  $\beta$ . Setting  $\beta_H = \beta_L$  implies an uninformative reporting system with no updating of beliefs. Hence, the outside party stays with prior belief  $\alpha < k$ , the threshold is not met, and  $B = 0$ . At the other extreme,  $\beta_H = 1$  and  $\beta_L = 0$  implies a perfectly informative system. In this case, the outside party's posterior belief equals 1 if  $r = g$ , implying the threshold is exceeded, and 0 if  $r = b$ , implying the threshold is not met. It follows that  $B = \alpha S = \alpha$ . Notably, assurance of a high state given a good report is a stronger condition than is necessary to meet the threshold. The firm can increase the probability of meeting the threshold by allowing some good reports to be generated in a low state. While this diminishes the posterior expectation given a good report, the expectation may still be sufficient to meet the threshold. Accordingly, in the absence of private information the firm maximizes the probability of a good report, subject to meeting the threshold. This is accomplished by setting  $\beta_H = 1$  and solving for  $\beta_L$  in the following expression:

$$\Pr(\theta = H|r = g) = \frac{\alpha \Pr(r = g|\theta = H)}{\alpha \Pr(r = g|\theta = H) + (1 - \alpha) \Pr(r = g|\theta = L)} = \frac{\alpha\beta_H}{\alpha\beta_H + (1 - \alpha)\beta_L} = k.$$

The optimal choice of  $\beta$ , with superscript "P" to indicate the "pure persuasion" benchmark, is

$$\beta_H^P = 1 \quad \text{and} \quad \beta_L^P = \frac{\alpha(1-k)}{k(1-\alpha)} \in (0,1),$$

implying an expected benefit in the pure persuasion benchmark case of  $B^P = \frac{\alpha}{k} > \alpha$ . While with perfect information the firm only meets the threshold with probability  $\alpha$ , an optimal reporting system improves the odds to  $\frac{\alpha}{k}$ . Both parties are rational and update consistent with Bayes' Rule, notwithstanding that the information provided by the firm's reporting system is slanted in a manner that serves the firm's interests.

The distribution over posterior beliefs (i.e., the outsider's expectation that the underlying state is high) generated by reports is as follows. The outsider has a posterior belief equal to  $k$  with probability  $\frac{\alpha}{k}$  and a posterior belief of 0 with probability  $\frac{k-\alpha}{k}$ . We note that these posterior beliefs satisfy the law of iterated expectations; i.e.,  $\frac{k-\alpha}{k} \times 0 + \frac{\alpha}{k} \times k = \alpha$ .<sup>6</sup>

### 3.3 Financial Reporting With Private Information

The possible receipt and discretionary disclosure of a private signal adds a second stage at which the firm makes a decision and the outside party updates beliefs. Accordingly, we solve the model by backward induction. Recall that the firm receives a benefit,  $S$ , if and only if  $E[\theta|r, m] \geq k$ . Having normalized the states at  $H = 1$  and  $L = 0$ , the above expectation is simply the posterior probability of  $\theta = H$  given a report  $r$  and message  $m$ , i.e.,

$$E[\theta|r, m] = \Pr(\theta = H|r, m).$$

Suppose the firm receives a private signal  $s$ . Since  $\gamma_H > \gamma_L$ , the probability of a high state is greater for a message  $m = h$  than for  $m = \emptyset$  and for a message  $m = \emptyset$  than for  $m = l$ . The lemma below follows immediately:

**Lemma 1** *The firm always discloses when  $s = h$  and never discloses when  $s = l$ .*

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<sup>6</sup> This is equivalent to the "Bayesian plausibility" requirement in Kamenica and Gentzkow (2011). In our case, we incorporate this requirement in our calculations of posterior beliefs using Bayes' Rule rather than explicitly including the requirement as an additional constraint in the optimization programs.

Moving back to the choice of parameters governing the financial reporting system  $\beta$ , there are several combinations of reports and messages that might maximize the joint probability of meeting the threshold.<sup>7</sup> Each combination gives rise to a constrained optimization program for which some  $\beta$  is optimal. As before, the firm wants to maximize the expected benefit  $B$  which equates to maximizing the probability of meeting the threshold. Below we specify the combinations and related programs where “ $D$ ” denotes discretionary disclosure:

$$\mathcal{P1}(D): \max_{\beta} \Pr(r = g, m = h) + \Pr(r = b, m = h) + \Pr(r = g, m = \emptyset)$$

$$\begin{aligned} s. t. \quad & E[\theta|r = g, m = h] \geq k, \quad E[\theta|r = b, m = h] \geq k, \\ & E[\theta|r = g, m = \emptyset] \geq k, \quad 1 \geq \beta_H \geq \beta_L \geq 0; \end{aligned}$$

$$\mathcal{P2}(D): \max_{\beta} \Pr(r = g, m = h) + \Pr(r = b, m = h)$$

$$\begin{aligned} s. t. \quad & E[\theta|r = g, m = h] \geq k, \quad E[\theta|r = b, m = h] \geq k, \\ & 1 \geq \beta_H \geq \beta_L \geq 0; \end{aligned}$$

$$\mathcal{P3}(D): \max_{\beta} \Pr(r = g, m = h) + \Pr(r = g, m = \emptyset)$$

$$\begin{aligned} s. t. \quad & E[\theta|r = g, m = h] \geq k, \quad E[\theta|r = g, m = \emptyset] \geq k, \\ & 1 \geq \beta_H \geq \beta_L \geq 0; \end{aligned}$$

$$\mathcal{P4}(D): \max_{\beta} \Pr(r = g, m = h)$$

$$s. t. \quad E[\theta|r = g, m = h] \geq k, \quad 1 \geq \beta_H \geq \beta_L \geq 0.$$

Elaborating on the composition of  $\mathcal{P1}(D)$ , the objective function is composed of the unconditional joint probability of report-message combinations including a good report and disclosure of a high signal, bad report and disclosure of a high signal, and good report and non-disclosure of a signal. The constraints ensure that the threshold is met for each combination and assumed properties of  $\beta$ . The first constraint (good report-high signal) will be slack, while at least one of the next two constraints (good report-no message or bad report-high signal) will

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<sup>7</sup> Recall that as per our discussion early on the firm sets the reporting system to maximize the expected benefit, which under our assumptions boils down to maximizing the probability of meeting the threshold.

bind, as both imply a lower probability of meeting the threshold. Each of the next two programs,  $\mathcal{P}2(D)$  and  $\mathcal{P}3(D)$ , considers two combinations of reports and messages while eliminating one of the potentially binding constraints in  $\mathcal{P}1(D)$ .  $\mathcal{P}4(D)$  considers only one combination while eliminating both of the potentially binding constraints in  $\mathcal{P}1(D)$ . Eliminating constraints enlarges the feasible regions, but reduces the set of report-message combinations that result in posterior beliefs at or above the threshold. Hence, *a priori* we cannot say which program solution will provide the highest probability and related expected benefit of meeting the threshold for a given set of exogenous parameters. Solutions to the programs are provided in the Appendix.

### 3.4 Characteristics of Optimal Financial Reporting Systems

We begin this section by identifying a set of conditions on model parameters that have a bearing on which of the solutions to the above programs dominates. These conditions lead to characterizations of optimal financial reporting systems. We further assess the impact of discretionary disclosure and, separately, the potential availability of private information by comparing the optimal financial reporting system with the solution to the pure persuasion game benchmark.

$$\textbf{Condition 1} \quad \gamma_H \geq \bar{g} \equiv \frac{1-\gamma_L k q}{q(2-\gamma_L q-k)}$$

We refer to the above condition as capturing private signal informativeness. Note that the lower bound,  $\bar{g}$ , on the probability of a high signal given a high state in Condition 1 is increasing in the probability of a high signal given a low state,  $\gamma_L$ . Either an increase in  $\gamma_H$  or a decrease in  $\gamma_L$  widens the spread between those probabilities, which naturally captures private signal informativeness. Accordingly, we classify private signals as more informative if Condition 1 is satisfied and as less informative otherwise.

$$\textbf{Condition 2} \quad \alpha > \frac{\gamma_L k}{\gamma_L k + \gamma_H (1-k)}$$

Prior beliefs are said to be optimistic if Condition 2 is satisfied and pessimistic otherwise.

**Proposition 1** *If Condition 1 is not satisfied (less informative private signals), then the firm's optimal financial reporting system is defined by  $1 = \beta_H^* > \beta_L^* > 0$  and the threshold  $k$  is met or exceeded for a report  $r = g$  (and either message  $m = h$  or  $m = \emptyset$ ).*

A less informative private signal implies a smaller shift in the outside party's posterior beliefs and, therefore, meeting the threshold requires a good public report. In this case, the solution of  $\mathcal{P}3(D)$  is globally optimal, i.e. the threshold is met following a good financial report irrespective of the message sent by the firm. In comparison with the solution to the benchmark pure persuasion game (equivalent to a completely uninformative private signal or  $q = 0$ ), it is optimal for the firm to choose a financial reporting system that generates a somewhat more informative good report. This is accomplished by reducing the probability of a good report in a low state,  $\beta_L^P > \beta_L^*$ , while holding constant the probability of a good report in a good state  $\beta_H^* = \beta_H^P = 1$ . While decreasing  $\beta_L$  implies a more informative good report, it also reduces the frequency of a good report, thereby lessening the unconditional probability of a report that induces a posterior expectation that meets the threshold. The former effect is necessary to allow the firm to meet the threshold with the combination of a good report and non-disclosure of a private signal (and exceed the threshold with a high signal). Although meeting the threshold with only a good report and a high signal as in  $\mathcal{P}4(D)$  would allow the firm to increase the frequency of a good report, the joint unconditional probability of just this combination is lower implying that the threshold would not be met as often.

When Condition 1 is not satisfied and private signal informativeness decreases further (through either a decrease in the probability of receiving a private signal  $q$ , an increase in the probability of a low signal given a high state,  $\gamma_L$ , or a decrease in the probability of a high signal given a high state,  $\gamma_H$ ), the financial reporting system becomes less informative; i.e.,  $\beta_L^*$  approaches  $\beta_L^P$ . As the next proposition establishes, increasing private signal informativeness to the point where Condition 1 is satisfied may lead to a more informative financial reporting system:

**Proposition 2** *When Condition 1 is satisfied (more informative private signals),*

- (i) *if Condition 2 is not satisfied, then the firm's optimal financial reporting system is defined by  $1 = \beta_H^{**} > \beta_L^{**} > 0$  and the threshold  $k$  is met for report  $r = g$  and message  $m = h$ .*
- (ii) *if Condition 2 is satisfied, then  $1 > \beta_H^{***} > \beta_L^{***} > 0$  and the threshold  $k$  is met for either report  $r = g$  or message  $m = h$ .*

Recall that Condition 1 is satisfied when private signals are more informative and

Condition 2 is satisfied when prior beliefs are optimistic. It is useful to compare the solution in part (i) of Proposition 2 to the solution in Proposition 1 in assessing the effect of satisfying Condition 1. A more informative private signal under program  $\mathcal{P}3(D)$  makes it more difficult to meet the threshold with the combination of a good report and non-disclosure of a signal. In other words, this combination implies a lower posterior belief, tightening the constraint on meeting the threshold for that combination due to a more informative low signal. As a consequence, the firm must choose a more informative but less frequent good report, which is accomplished by reducing the probability of a good report in a low state. However, the firm can do better in program  $\mathcal{P}4(D)$ , where a good report and a more informative high signal imply a higher posterior belief. This combination allows the firm to relax the constraint on meeting the threshold by choosing a less informative but more frequent good report, achievable by increasing the probability of a good report in a low state in comparison to the benchmark pure persuasion game, i.e.,  $\beta_L^P < \beta_L^{**}$ .

Less informative private signal	Threshold $k$ met IFF $r = g$ Chooses $\beta^*$ such that $1 = \beta_H^* = \beta_H^P$ and $0 < \beta_L^* < \beta_L^P$	
More informative private signal	Threshold $k$ met IFF $r = g$ OR $m = h$ Chooses $\beta^{***}$ such that $1 = \beta_H^P > \beta_H^{***}$ and $0 < \beta_L^{***} < \beta_L^P$	Threshold $k$ met IFF $r = g$ AND $m = h$ Chooses $\beta^{**}$ such that $1 = \beta_H^P = \beta_H^{**}$ and $0 < \beta_L^P < \beta_L^{**}$
	Optimistic beliefs	Pessimistic beliefs

**Figure 2**

Firm's choice of financial reporting system defined by  $\beta_H$  and  $\beta_L$ . Stars indicate optima as described in Propositions 1 (\*), 2.i (\*\*), and 2.ii (\*\*\*);  $r \in \{g, b\}$  is the public report;  $m \in \{s, \emptyset\}$  is the discretionary message based on the private signal  $s \in \{h, l\}$ ; and  $\beta_H$  ( $\beta_L$ ) is the probability of  $r = g$  conditional on the state being high (low).

When Conditions 1 and 2 are satisfied, having both optimistic prior beliefs and a more informative private signal makes it possible for the firm to meet the threshold with a combination of a bad report and high private signal. This is achieved by reducing the probability of a good report in a good state such that a bad report no longer implies a low state with certainty. Accordingly, under part (ii), the firm does best in program  $\mathcal{P}1(D)$  where the threshold is met by



a combination of good report and any message or a bad report and a high message. In this case, the firm also reduces the probability of a good report in a low state in comparison to the benchmark pure persuasion game; i.e.,  $\beta_H^{***} < \beta_H^P = 1$  and  $\beta_L^{***} < \beta_L^P$ . While the firm can benefit from having a high message, in order to benefit when a low message is sent, it is crucial to make the bad report less than fully informative, which implies setting  $\beta_H < 1$ .

Figure 2 summarizes the results of Propositions 1 and 2. As we would anticipate, public financial reports and messages of private information are partial substitutes. Under pessimistic prior beliefs, less (more) informative private signals imply the choice of a more (less) informative financial reporting system i.e.,  $\beta_L^{**} > \beta_L^*$ . The implication of more informative private signals for the informativeness of the financial reporting system in the remaining case of optimistic prior beliefs requires a measure of informativeness that encompasses both parameters  $\beta_H$  and  $\beta_L$ . For this case, we resort to the variance of conditional expectations (VCE) to show that less (more) informative private signals again imply the choice of a more (less) informative financial reporting system.<sup>8</sup>

### 3.5 Welfare Implications

Our analysis focuses on the expected utility of the firm, but there are several settings in which the firm's expected utility is a sufficient statistic for certain broader welfare orderings. In some instances, we might view the outside party as an intermediary who assesses compliance with a threshold based on pre-established rules; e.g., an auditor for whom criteria for an unqualified opinion are set by generally accepted auditing standards. For example, a sufficiently low posterior expectation of firm value could lead to a going concern qualification, which might then trigger a loss. While the auditor may have little, if any, flexibility in applying such rules, he would prefer that the client achieves the threshold under the assumption that the likelihood of a continuing engagement is advanced by an unqualified opinion. In this case, the firm's and the auditor's preferences with respect to the choice of a financial reporting system are aligned, implying total welfare corresponding to the firm's expected benefits. A bond rating organization obliged to follow a fixed protocol might benefit more from a firm meeting criteria on expected future payoffs for a high rating that generates greater interest by traders who subscribe to the

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<sup>8</sup> See the appendix for the proof. Note that VCE and equivalent measures have been used in prior studies as measures of information content (e.g., Friedman et al. 2015).

service. In other instances, the outside party might be indifferent to whether a threshold is met. An example here could be a bank examiner who assesses the adequacy of loan loss reserves.<sup>9</sup> In all of these cases, the firm's expected utility is a sufficient statistic for the welfare of the firm-outside party pair.

Other examples of an outside party include a competitive lender or investor for whom the threshold takes the form of a required expected return. If the outside party represents a set of competitive lenders or investors, then the firm's expected utility would also serve as a sufficient statistic for the joint utility of the firm and the outsider(s). For example, consider an entrepreneur who must raise capital to implement a project that would otherwise be lost at some cost or impose a loss in expected utility on the entrepreneur. Given that the entrepreneur can exploit the competition amongst investors to extract surplus in excess of the outside party's required return, the welfare ordering reduces to the ordering implied by the expected benefits to the firm as depicted by our propositions.<sup>10</sup> Note that in this example, the entrepreneur derives a further benefit beyond that of meeting the threshold in the form of expected returns in excess of the outside parties required return.

### 3.6 Implied Liberal Bias in Financial Reporting

We relate our results expressed in terms of  $\beta$  to a bias proxy, denoted,  $\chi$ , as in Friedman et al. (2015) through the following transformation of variables:

$$\chi \equiv \frac{1-\beta_L-\beta_H}{2}.$$

A positive value of  $\chi$  connotes a conservative bias while a negative value connotes a liberal bias. The implied biases corresponding to the solutions in the benchmark pure persuasion case and Propositions 1 and 2 are liberal consistent with the tendency in all cases to increase the frequency of good reports while reducing their informativeness in order to produce the highest joint unconditional probability of meeting the threshold. Only the solution to Proposition 2 (i) includes a liberal bias greater than that in the pure persuasion game; i.e.,  $\chi^{**} < \chi^P$ . This is because with pessimistic prior beliefs, the firm relies on both a good financial report and a more

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<sup>9</sup> Ignored in these examples is the welfare of those who rely on the assessments made by intermediaries such as those above.

<sup>10</sup> There is a subtlety here in that the deadweight costs or loss in utility that the entrepreneur avoids by meeting the threshold dominates the inefficiency implied by overinvestment in comparison to a perfectly informative reporting system.

informative high private signal to meet the threshold. The latter allows the firm to further increase the unconditional probability of a good report by more liberally biasing the reporting system than in the pure persuasion game. In the other two cases, discretionary disclosure of private signals leads to less liberal biasing of the financial reporting system; i.e.,  $\chi^* > \chi^P$  and  $\chi^{***} > \chi^P$ . Supposing that regulators such as the SEC and FASB may seek on general principles to induce more informative financial reporting, then this is advanced by less liberal (equivalently, more conservative) reporting in the sense of reducing the probability of a good report in a low state; i.e., decreasing  $\beta_L$ .

## 4. Extensions

### 4.1 Are Firms Better Off with Private Information?

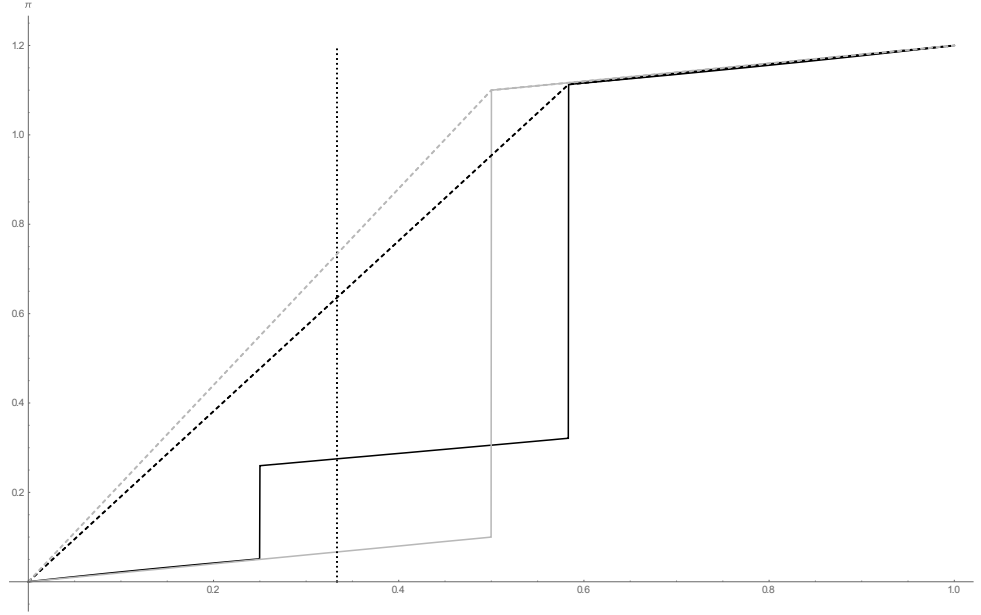
Comparing the expected benefit (probability of meeting the threshold multiplied by the firm's benefit of 1 when meeting the threshold) corresponding to the solution in Propositions 1,  $B^*$ , with that in the pure persuasion game,  $B^P$ , we see that the addition of a less informative private signal to financial reports lowers the expected benefit. The firm still expects to do better, though, than it would if it provided a perfectly informative financial reporting system:

**Corollary 1** *Suppose Condition 1 is not satisfied. Then,  $B^P > B^* > \alpha$ .*

The proof is omitted as it follows directly from the comparison of the expected benefits at  $\beta^P$  and  $\beta^*$ . To explain the driving forces behind this result let

$$\mu \equiv E[\theta|r] \quad \text{and} \quad \Pi(\mu) = E_m[\pi(\mu, m)].$$

As shown in Kamenica and Gentzkow (2011) the expected benefit from the optimal reporting system depends on the concave closure of  $\Pi(\mu)$  when the firm might have access to private information, and depends on the concave closure of  $\pi(\mu)$  when the firm surely lacks such access. Kamenica and Gentzkow (2011) provide this result in terms of a receiver with beliefs that the sender does not know when designing the reporting system. In our setting, the firm's potential receipt and disclosure of private information causes it to be uncertain of the outsider's message-dependent beliefs at the time when the firm chooses  $\beta$ . As defined above,  $\pi(\mu)$  has a jump of  $S$  at the point where the posterior expectation based on the firm's report,  $\mu(r)$ , equals



**Figure 3**

Comparison of expected benefits in the pure persuasion benchmark (gray) and when the firm may have private information (black) when Condition 1 is not satisfied (Corollary 1). Solid lines represent firm benefits. Dashed lines are concave closures, and the vertical dotted line marks  $\mu = \alpha$ . Parameters are set as  $S = 1$ ,  $\sigma = \frac{1}{5}$ ,  $k = \frac{1}{2}$ ,  $\alpha = \frac{1}{3}$ ,  $\gamma_L = \frac{1}{4}$ ,  $\gamma_H = \frac{3}{4}$ , and  $q = \frac{1}{2}$ .

the threshold,  $k$ .  $\Pi(\mu)$  has a jump of  $S \cdot \Pr(m = h)$  at the point where the posterior expectation based on the firm's report,  $\mu(r)$ , combined with a high message,  $m = h$ , equals  $k$ , and a further jump of  $S \cdot \Pr(m = \emptyset)$  at the point where the posterior,  $\mu(r)$ , combined with a null message,  $m = \emptyset$ , equals  $k$ .<sup>11</sup> The total vertical distance of the two jumps in  $\Pi(\mu)$  is equal to the vertical jump in  $\pi(\mu)$ , since the first step is  $S \cdot \Pr(m = h)$ , the second step is  $S \cdot \Pr(m = \emptyset)$ , and  $\Pr(m = \emptyset) + \Pr(m = h) = 1$ .

Similar to Kamenica and Gentzkow (2011) the maximum expected payoff, which is achieved with the optimal reporting system, is the concave closure of  $\Pi(\mu)$  or  $\pi(\mu)$  (depending on whether the firm might have private information) evaluated at the prior belief  $\alpha$ . That is, our primary concern from an expected benefit standpoint is the value of the concave closure of the payoff function, evaluated at  $\alpha$ . As illustrated by the numerical example in Figure 3, the concave closure of  $\pi(\mu)$  is above the concave closure of  $\Pi(\mu)$  evaluated at the prior belief,  $\alpha$ , when Condition 1 is not satisfied. This implies that the firm's expected payoff is always lower when it has potential access to private information, compared to the case when it is

<sup>11</sup> Both  $\Pi(\mu)$  and  $\pi(\mu)$  include a linear component as well, but it is irrelevant for this discussion.

known to be uninformed.

Similar to the ordering of expected benefits in Corollary 1 for the case described in Proposition 1, expected benefits in both cases considered in Proposition 2 are lower than in the benchmark pure persuasion game:

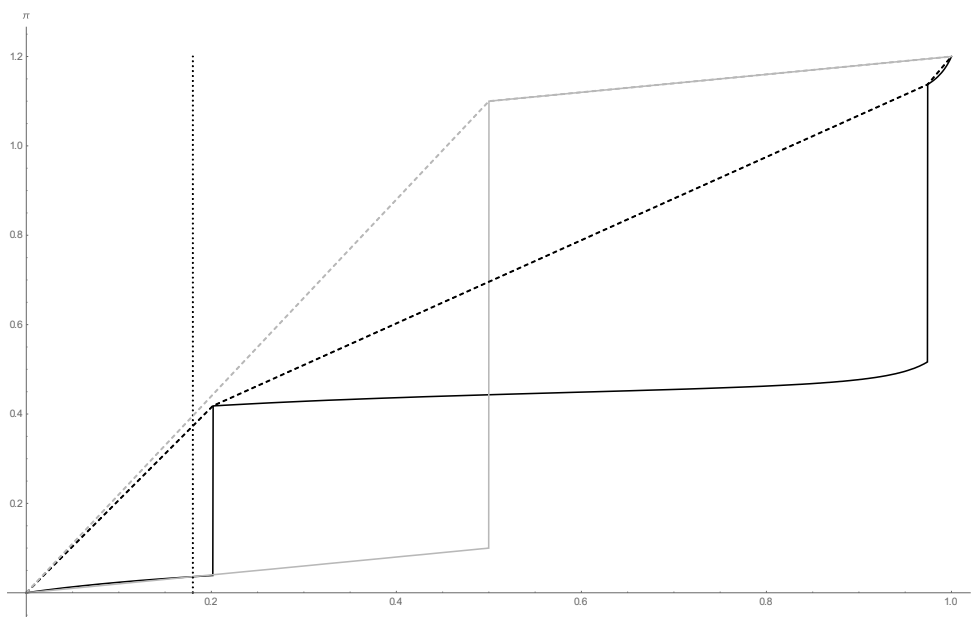
**Corollary 2** *Suppose Condition 1 is satisfied.*

(i) *If Condition 2 is not satisfied, then  $B^P > B^{**} > \alpha$ .*

(ii) *If Condition 2 is satisfied, then  $B^P > B^{***} > \alpha$ .*

The proofs are omitted and their intuition follows a similar logic to that of Corollary 1. Numerical examples in Figures 3, 4 and 5 illustrate parts (i) and (ii) of Corollary 2. It is evident from Corollaries 1 and 2 that, in the context of our model, discretionary disclosure of private information does not enhance the firm's ability to meet the threshold over what the firm could achieve with the financial reporting system alone, absent potential receipt of private information.

These results further imply that if the firm had control over the private information, it would choose never to receive such information (i.e., set  $q = 0$ ) or choose a completely



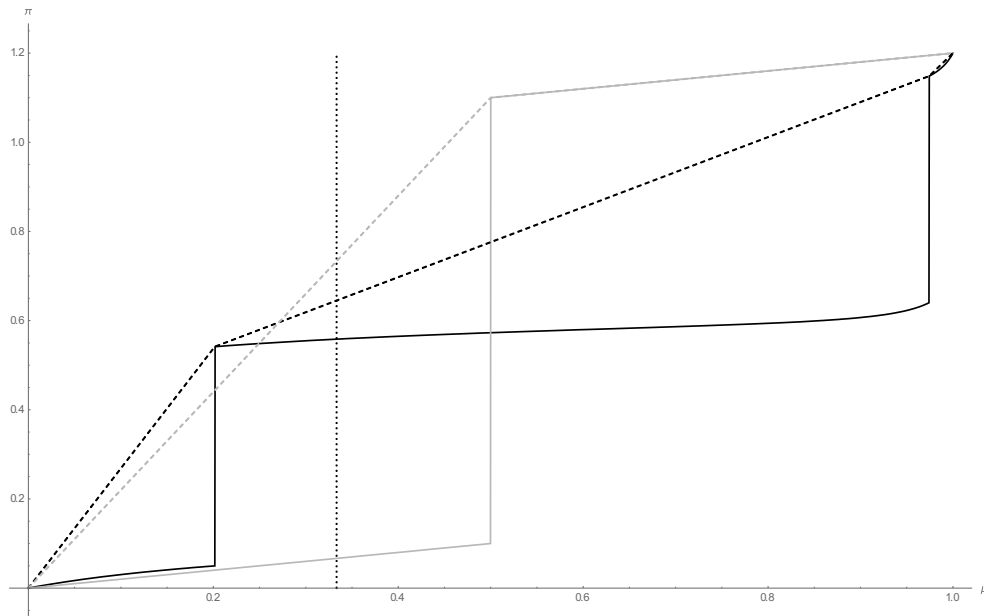
**Figure 4**

Comparison of expected benefits in the pure persuasion benchmark (gray) and when the firm may have private information (black) when Condition 1 is satisfied but Condition 2 is not (Corollary 2(i)). Solid lines represent firm benefits. Dashed lines are concave closures, and the vertical dotted line marks  $\mu = \alpha$ . Parameters are set as  $S = 1$ ,  $\sigma = \frac{1}{5}$ ,  $k = \frac{1}{2}$ ,  $\alpha = \frac{18}{100}$ ,  $\gamma_L = \frac{1}{4}$ ,  $\gamma_H = \frac{99}{100}$ , and  $q = \frac{99}{100}$ .

uninformative private information system so that the outsiders ignore the message (i.e., set  $\gamma_H = \gamma_L$ ). However, an ability to forestall the receipt of private information would seem to be impossible given all of the ways in which information may arrive. It would appear to be similarly impossible to design commitments not to disclose information when meeting a crucial threshold is at risk.

## 4.2 Non-disclosure Equilibrium

In the preceding section, we showed that in the absence of control over the arrival of private information the firm is stuck in a less desirable equilibrium. In this subsection, we discuss the implications of assuming no further benefit to reporting beyond meeting the threshold (i.e.,  $\sigma = 0$ ). Interestingly, when  $\sigma = 0$ , there exists an equilibrium in which the firm chooses never to disclose private information. When  $\sigma = 0$  the threshold decision rule of our model leads to regions of indifference with respect to the disclosure of a high signal. Depending on the tie rule it may be sequentially rational for the firm to choose not to disclose a high signal in programs where the constraint on the combination of a good report and a high private signal



**Figure 5**

Comparison of expected benefits in the pure persuasion benchmark (gray) and when the firm may have private information (black) when Conditions 1 and 2 are satisfied (Corollary 2(ii)). Solid lines represent firm benefits. Dashed lines are concave closures, and the vertical dotted line marks  $\mu = \alpha$ . Parameters are set as  $S = 1$ ,  $\sigma = \frac{1}{5}$ ,  $k = \frac{1}{2}$ ,  $\alpha = \frac{1}{3}$ ,  $\gamma_L = \frac{1}{4}$ ,  $\gamma_H = \frac{99}{100}$ , and  $q = \frac{99}{100}$ .

results in a posterior belief strictly exceeding the threshold; i.e., the constraint corresponding to this combination is slack. This is the case in programs  $\mathcal{P}3(D)$  and  $\mathcal{P}1(D)$  which define the globally optimal solutions in Propositions 1 and 2(i). Although the constraint based on the combination of a good report and high signal is binding in program  $\mathcal{P}4(D)$ , this constraint can be relaxed without a different program replacing  $\mathcal{P}4(D)$  in the case of Proposition 2(ii). Since this exhausts the set of parameters relevant to the firm's choices, it is evident that an equilibrium exists in which the firm does not disclose any signal.

To see the intuition, suppose that the outside party believed the firm would never disclose a private signal. Now assume that the firm receives a high signal. Would the firm disclose? In order to sustain the outside party's belief that it would not disclose a high signal if received, the firm must choose the same financial reporting system as in the pure persuasion game. For a good report, disclosing the high signal would induce a posterior expectation by the outside party in excess of the threshold which provides no explicit further benefit to the firm. For a bad report, disclosing a high signal is moot since a bad report implies a low state for sure in the pure persuasion game. Since *ex ante* the solution to the pure persuasion game at least weakly dominates the solution under discretionary disclosure, the firm has no incentive to defect at either stage.

A natural question is whether these equilibria can be ordered from the firm's point of view. As we showed, in each of the cases represented in Propositions 1 and 2, the expected benefit to the firm is greater in the benchmark pure persuasion game than in any case with discretionary disclosure of private signals. It follows that, in the absence of an additional marginal benefit from disclosure of a high signal *per se* (i.e.,  $\sigma = 0$ ), a non-disclosure equilibrium exists and dominates from the firm's point of view. While we find this result interesting, the assumption of no further benefit to disclosure of a high signal is not descriptive of situations that firms actually face given the many other roles that have been ascribed to discretionary disclosure. Any marginal benefit to disclosure of a high private signal beyond that of meeting a crucial threshold, in and of itself, no matter how negligible, suffices to eliminate this equilibrium, notwithstanding that the firm may be better off with no disclosure.

### **4.3 The Value of Discretion to Disclose Private Information**

We next consider whether the option to disclose or not disclose is beneficial to the firm.

To do so, we solve for the optimal financial reporting system design through a series of programs similar to those in the previous section except that both low and high signals are disclosed. We refer to this as a *mandatory* disclosure setting, as the firm is assumed to disclose its private information, and denote solutions and optimization problems here by "M".

$$\mathcal{P}1(M): \max_{\beta} \Pr(r = g, m = h)$$

$$s. t. \quad E[\theta|r = g, m = h] \geq k, \quad 1 \geq \beta_H \geq \beta_L \geq 0.$$

$$\mathcal{P}2(M): \max_{\beta} \Pr(r = g, m = h) + \Pr(r = g, m = \emptyset)$$

$$s. t. \quad E[\theta|r = g, m = h] \geq k, \quad E[\theta|r = g, m = \emptyset] \geq k, \\ 1 \geq \beta_H \geq \beta_L \geq 0;$$

$$\mathcal{P}3(M): \max_{\beta} \Pr(r = g, m = h) + \Pr(r = b, m = h)$$

$$s. t. \quad E[\theta|r = g, m = h] \geq k, \quad E[\theta|r = b, m = h] \geq k, \\ 1 \geq \beta_H \geq \beta_L \geq 0;$$

$$\mathcal{P}4(M): \max_{\beta} \Pr(r = g, m = h) + \Pr(r = b, m = h) + \Pr(r = g, m = \emptyset)$$

$$s. t. \quad E[\theta|r = g, m = h] \geq k, \quad E[\theta|r = b, m = h] \geq k, \\ E[\theta|r = g, m = \emptyset] \geq k, \quad 1 \geq \beta_H \geq \beta_L \geq 0;$$

$$\mathcal{P}5(M): \max_{\beta} \Pr(r = g, m = h) + \Pr(r = g, m = \emptyset) + \Pr(r = g, m = l)$$

$$s. t. \quad E[\theta|r = g, m = h] \geq k, \quad E[\theta|r = g, m = \emptyset] \geq k, \\ E[\theta|r = g, m = l] \geq k, \quad 1 \geq \beta_H \geq \beta_L \geq 0$$

$$\mathcal{P}6(M): \max_{\beta} \Pr(r = g, m = h) + \Pr(r = g, m = \emptyset)$$

$$+ \Pr(r = g, m = l) + \Pr(r = b, m = h)$$

$$s. t. \quad E[\theta|r = g, m = h] \geq k, \quad E[\theta|r = g, m = \emptyset] \geq k, \\ E[\theta|r = g, m = l] \geq k, \quad E[\theta|r = b, m = h] \geq k, \\ 1 \geq \beta_H \geq \beta_L \geq 0.$$



**Proposition 3** *Suppose Condition 1 is not satisfied, with  $\gamma_H$  sufficiently low, i.e.,  $\gamma_H < \min\{\bar{g}, g_o, g_{oo}\}$ . Then, the firm strictly prefers discretion over mandatory disclosure.*

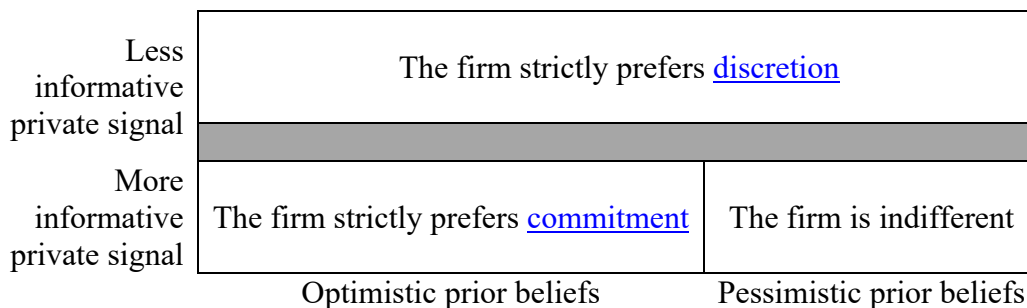
The applicable discretionary disclosure case for this parameterization is depicted in Proposition 1. Under mandatory disclosure, a low private signal is no longer pooled with non-receipt of a signal. As a consequence, a good report need not be as informative as under discretionary disclosure in order for the posterior beliefs following a combination of a good report and non-disclosure to meet the threshold. However, a good report that is only sufficiently informative to meet the threshold for that combination, when combined with a low signal, will not meet the threshold. If under mandatory disclosure the firm sought to meet the threshold for both non-receipt of a signal and a low signal, then the effect of having to compensate for a low signal in choosing a reporting system implies a worse solution than for  $\mathcal{P}3(D)$ . In the proof, we show that the solution to  $\mathcal{P}3(D)$  exceeds to solution to all programs  $\mathcal{P}1(M) - \mathcal{P}6(M)$ . Hence, discretion in this case is valuable to the firm.

**Proposition 4** *Suppose Condition 1 is satisfied, with  $\gamma_H > \max\{\bar{g}, g^o, g^{oo}\}$ . Then,*

- (i) if Condition 2 is not satisfied, then the firm is indifferent between discretion and mandatory disclosure.*
- (ii) if Condition 2 is satisfied, then the firm strictly prefers mandatory disclosure to discretion.*

The applicable discretionary disclosure cases for these parameterizations are  $\mathcal{P}4(D)$  and  $\mathcal{P}1(D)$ , respectively. In part (i), we show that  $\mathcal{P}1(M)$  is globally optimal. Since it corresponds to  $\mathcal{P}4(D)$  by considering only the combination of a good report and high signal in meeting the threshold, then the solutions are identical implying indifference by the firm between discretionary and mandatory disclosure. As for part (ii), we show that  $\mathcal{P}4(M)$  brings a higher expected benefit to the firm than the globally optimal program with discretion,  $\mathcal{P}1(D)$ . While they both consider combinations of a good report and high signal, bad report and high signal, and high report and non-disclosure, the former does not pool non-receipt of a signal with a low signal, thereby making it possible to meet the threshold with a less informative but more frequent

good report. Hence, the firm strictly prefers mandatory disclosure to discretion. Figure 6 depicts the regions in which the firm prefers either of the two regimes.



**Figure 6**  
Firm's preference over regimes

Although interesting as a benchmark in appreciating the consequences of discretion, mandatory disclosure may not be a realistic option given that the costs of monitoring compliance and enforcing penalties for non-compliance when a firm's receipt of information is uncertain are likely to be prohibitively high.

## 5 Conclusion

We consider the effects of discretionary disclosure of private information on financial reporting system design choices. Our model is an extension of Bayesian persuasion games in which a sender makes an *ex ante* choice of a reporting system with the objective of maximizing the posterior expectation of meeting a threshold set by a receiver and upon which the sender's welfare depends. The sender in the context of our model is a firm and the receiver is an outside party such as auditor, credit rater, lender, or other certifying agency. Failure to reach the bar for an unqualified opinion, an investment-grade debt rating, a key covenant, or any of myriad certification requirements, implies a significant loss to the firm. While a perfectly informative reporting system is assumed to be feasible at no cost, the firm can do better with a less informative system that enhances the firm's odds of generating posterior beliefs that just meet the threshold. The firm's optimal design in such a setting can be viewed as a liberal or aggressive set of accounting policies.

The prospect of receiving private information, over which disclosure by the firm is

discretionary, induces the firm to change the properties of its financial reporting system. When private signals are less informative, the firm directs its financial reporting system toward providing more informative favorable reports. This is because such reports have to raise the posterior beliefs sufficiently to offset the negative influence of the potential non-disclosure of a private signal, given that such non-disclosure may be due to an unfavorable private signal or no private signal having been received. When private signals are more informative and prior beliefs are pessimistic, then the firm would choose less informative favorable reports, anticipating that disclosure of a sufficiently favorable private signal would compensate for the effect of an unfavorable report on posterior expectations. The firm's financial reporting system choices in the remaining case of more informative private signals and optimistic prior beliefs are more complex involving both less informative favorable reports and more informative unfavorable reports. Constructively, financial reports and private signals are partial substitutes. Less informative private signals in general imply a choice of more informative financial reporting systems.

Generally speaking, in settings where outside parties employ only a threshold decision rule, discretionary disclosure of private information provides no benefit to the firm beyond that achievable through a judicious choice of a public financial reporting system. While likely to be less descriptive of situations that firms may face, absent a marginal further benefit to disclosing favorable private signals *per se*, an alternative equilibrium exists in which the firm does not disclose even those signals which would increase the likelihood of meeting or exceeding the threshold. Comparing regimes with discretionary and mandatory disclosure of private information, there are conditions under which the firm may prefer one or the other. In particular, a combination of optimistic prior beliefs and highly informative private signals implies a preference for mandatory disclosure. Although useful as a theoretical benchmark, implementing mandatory disclosure would require monitoring of the receipt of private information and penalties for non-compliance, which may be infeasible or, at best, very costly.

Our model is highly stylized to focus on but one tension the firm faces in choosing the properties of its reporting system. Nonetheless, we believe that meeting a crucial threshold could be an overriding concern for some firms during time spans long enough to influence financial accounting policy decisions. The flexibility afforded firms by accounting standards in choosing accounting policies constitutes a natural device for firms to employ in seeking to meet thresholds or otherwise influence the beliefs held by financial statement users.

Giving some thought to empirical applications, we note that in 2005 the S.E.C. liberalized its “quiet period” policies to allow more information to be communicated for certain organizations following the filing of a registration statement. For IPOs this period is often referred to as a “cooling-off period.” In the context of our study, such a period, if enforced, may serve as a commitment device that benefits the firm, notwithstanding that its effect may be to diminish the informativeness of prospectuses. Relaxing these policies may have the opposite effect suggesting a natural experiment to test our predictions may be feasible. There is some prospect that these policies may be further liberalized or even eliminated given the commonly held view echoed by Fortune magazine’s 2011 feature article “It’s time to kill the IPO quiet period.” Given that the ability to meet a crucial threshold may only be present for some firms, there is scope for cross-sectional differences that could contribute to the power of one’s tests.

## Appendix

**Proof of Lemma 1:** The firm discloses  $s = h$  whenever:

$$\begin{aligned}\Delta_h &\equiv E[\theta|r, m = h] - E[\theta|r, m = \emptyset] \\ &= \Pr(\theta = H|r, m = h) - \Pr(\theta = H|r, m = \emptyset) \\ &\geq 0, \quad \forall r = g, b.\end{aligned}$$

It is straightforward to verify that, because  $\gamma_H > \gamma_L$  by assumption,  $\Pr(\theta = H|r, m = h) \geq \Pr(\theta = H|r, m = \emptyset)$ ,  $\forall r = g, b$ . Hence, the firm discloses  $s = h$ . Next, we show that the firm withholds  $s = l$ :

$$\begin{aligned}\Delta_l &\equiv E[\theta|r, m = l] - E[\theta|r, m = \emptyset] \\ &= \Pr(\theta = H|r, m = l) - \Pr(\theta = H|r, m = \emptyset) \\ &\leq 0, \quad \forall r = g, b.\end{aligned}$$

It is straightforward to verify that, because  $\gamma_H > \gamma_L$  by assumption,  $\Pr(\theta = H|r, m = l) \leq \Pr(\theta = H|r, m = \emptyset)$ ,  $\forall r = g, b$ . Hence, the firm withholds  $s = l$ .

**Proof of Proposition 1:**  $\mathcal{P}1(D)$  can be rewritten as:

$$\begin{aligned}\max_{\beta_H, \beta_L} & \alpha\beta_H + (1 - \alpha)\beta_L + (1 - p)(\alpha(1 - \beta_H)\gamma_H + (1 - \alpha)(1 - \beta_L)\gamma_L) \\ \text{s. t.} & E[\theta|r = g, m = h] \geq k, \\ & E[\theta|r = g, m = \emptyset] \geq k, \\ & E[\theta|r = b, m = h] \geq k, \\ & 1 \geq \beta_H \geq \beta_L \geq 0.\end{aligned}$$

The first condition is slack whenever either the second, or the third condition are satisfied (because  $E[\theta|r = g, m = h] \geq E[\theta|r = g, m = \emptyset]$  and  $E[\theta|r = g, m = h] \geq E[\theta|r = b, m = h]$ ). Therefore, the Lagrangian is

$$\begin{aligned}\mathcal{L}_1 &= \alpha\beta_H + (1 - \alpha)\beta_L + q(\alpha(1 - \beta_H)\gamma_H + (1 - \alpha)(1 - \beta_L)\gamma_L) \\ &+ \mu_1(E[\theta|r = g, m = \emptyset] - k) + \mu_2(E[\theta|r = b, m = h] - k) \\ &+ \mu_3(1 - \beta_H) + \mu_4(\beta_H - \beta_L) + \mu_5\beta_L.\end{aligned}$$

The Karush-Kuhn-Tucker stationarity conditions are:

$$\begin{aligned}\frac{\partial \mathcal{L}_1}{\partial \beta_H} &= \alpha(1 - q\gamma_H) \\ &+ \mu_1 \frac{\partial E[\theta|r=g, m=\emptyset]}{\partial \beta_H} + \mu_2 \frac{\partial E[\theta|r=b, m=h]}{\partial \beta_H} \\ &- \mu_3 + \mu_4 = 0\end{aligned}\tag{1}$$

$$\begin{aligned}\frac{\partial \mathcal{L}_1}{\partial \beta_L} &= (1 - \alpha)(1 - q\gamma_L) \\ &+ \mu_1 \frac{\partial E[\theta|r=g, m=\emptyset]}{\partial \beta_L} + \mu_2 \frac{\partial E[\theta|r=b, m=h]}{\partial \beta_L} \\ &- \mu_4 + \mu_5 = 0,\end{aligned}\tag{2}$$

the Karush-Kuhn-Tucker feasibility conditions are:

$$E[\theta|r = g, m = \emptyset] - k \geq 0\tag{3}$$

$$E[\theta|r = b, m = h] - k \geq 0\tag{4}$$

$$1 - \beta_H \geq 0\tag{5}$$

$$\beta_H - \beta_L \geq 0,\tag{6}$$

$$\beta_L \geq 0\tag{7}$$

$$\mu_i \geq 0, \quad i = 1, 2, 3, 4, 5.\tag{8}$$

and the Karush-Kuhn-Tucker complementarity slackness conditions are:

$$\mu_1(E[\theta|r = g, m = \emptyset] - k) = 0 \quad (9)$$

$$\mu_2(E[\theta|r = b, m = h] - k) = 0 \quad (10)$$

$$\mu_3(1 - \beta_H) = 0 \quad (11)$$

$$\mu_4(\beta_H - \beta_L) = 0 \quad (12)$$

$$\mu_5\beta_L = 0. \quad (13)$$

With five complementarity slackness conditions there are  $2^5 = 32$  cases. We can immediately rule out:

- All cases with  $\mu_3 > 0$  (so  $\beta_H = 1$ ) because if  $\beta_H = 1$ , then  $E[\theta|r = b, m = \emptyset] = 0 < k$  which is a contradiction;
- All cases with  $\mu_1 > 0, \mu_5 > 0$  (so  $E[\theta|r = g, m = \emptyset] = k, \beta_L = 0$ ) because if  $\beta_L = 0$ , then  $E[\theta|r = g, m = \emptyset] = 1 > k$  which is a contradiction;
- All cases with  $\mu_4 > 0$  ( $\beta_H = \beta_L$ ), because if  $\beta_H = \beta_L$ , then  $E[\theta|r = g, m = \emptyset] = E[\theta|r = b, m = \emptyset] < k$  which is a contradiction.
- All cases with  $\mu_2 = 0, \mu_3 = 0$  (so  $E[\theta|r = b, m = h] > k$  and  $1 > \beta_H$ ), because then  $\frac{\partial \mathcal{L}_1}{\partial \beta_H} = \alpha(1 - q\gamma_H) + \mu_1 \frac{\partial E[\theta|r=g, m=\emptyset]}{\partial \beta_H} + \mu_4 > \mu_1 \frac{\partial E[\theta|r=g, m=\emptyset]}{\partial \beta_H} \geq 0$ , implying  $\beta_H = 1$  which is a contradiction;
- All cases with  $\mu_1 = 0, \mu_4 = 0$  (so  $E[\theta|r = g, m = \emptyset] > k$  and  $\beta_H > \beta_L$ ), because then  $\frac{\partial \mathcal{L}_1}{\partial \beta_L} = (1 - \alpha)(1 - q\gamma_L) + \mu_2 \frac{\partial E[\theta|r=b, m=h]}{\partial \beta_L} + \mu_5 > \mu_2 \frac{\partial E[\theta|r=b, m=h]}{\partial \beta_L} \geq 0$ , implying  $\beta_L = 1 \geq \beta_H$  which is a contradiction;

We are left with only one case to consider:

- $\mu_1 > 0, \mu_2 > 0, \mu_3 = 0, \mu_4 = 0, \mu_5 = 0, E[\theta|r = g, m = \emptyset] = k, E[\theta|r = b, m = h] = k, 1 > \beta_H > \beta_L > 0$

We solve the four equations below

$$E[\theta|r = g, m = \emptyset] - k = 0$$

$$E[\theta|r = b, m = h] - k = 0$$

$$\alpha(1 - q\gamma_H) + \mu_1 \frac{\partial E[\theta|r=g, m=\emptyset]}{\partial \beta_H} + \mu_2 \frac{\partial E[\theta|r=b, m=h]}{\partial \beta_H} = 0$$

$$(1 - \alpha)(1 - q\gamma_L) + \mu_1 \frac{\partial E[\theta|r=g, m=\emptyset]}{\partial \beta_L} + \mu_2 \frac{\partial E[\theta|r=b, m=h]}{\partial \beta_L} = 0$$

and get

$$\beta_H = \frac{(\gamma_H \alpha(1-k) - \gamma_L k(1-\alpha))(1-q\gamma_L)}{\alpha(\gamma_H - \gamma_L)(1-k)}$$

$$\beta_L = \frac{(\gamma_H \alpha(1-k) - \gamma_L k(1-\alpha))(1-q\gamma_H)}{k(\gamma_H - \gamma_L)(1-\alpha)}$$

$$\mu_1 = \frac{\Gamma(\gamma_H(1-k)\alpha - \gamma_L k(1-\alpha))(\gamma_L k + \gamma_H(1-k - \gamma_L q))}{(\gamma_H - \gamma_L)^2(1-k)^2 k^2}$$

$$\mu_2 = \frac{\Gamma\gamma_H\gamma_L(k - \alpha(1 - q(\gamma_H k + \gamma_L(1-k))) - \gamma_L k q)}{(\gamma_H - \gamma_L)^2(1-k)^2 k^2}$$

where  $\Gamma \equiv (1 - \gamma_H q)(1 - \gamma_L q)$ . It is straightforward to verify that this case is feasible in a sense that  $1 > \beta_H > \beta_L > 0$  and  $\mu_1 > 0, \mu_2 > 0$  whenever  $\alpha > \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$ . For future reference,

- if  $\alpha > \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$ , then
 
$$1 > \frac{(\gamma_H \alpha(1-k) - \gamma_L k(1-\alpha))(1-q\gamma_L)}{\alpha(\gamma_H - \gamma_L)(1-k)} = \beta_H$$

$$\begin{aligned}
&> \beta_L \\
&= \frac{(\gamma_H \alpha (1-k) - \gamma_L k (1-\alpha))(1-q\gamma_H)}{k(\gamma_H - \gamma_L)(1-\alpha)} \\
&> 0
\end{aligned}$$

and the value of  $\mathcal{P1}(D)$  is:

$$\frac{M_A \cdot M_B + M_C}{k(1-k)(\gamma_H - \gamma_L)};$$

where  $M_A \equiv \alpha(1 - (\gamma_H(1-k) + \gamma_L k)q)$ ,  $M_B \equiv -(\gamma_L k + \gamma_H(1-k - \gamma_L q))$  and  $M_C \equiv \gamma_L k(1 - q(\gamma_L k + \gamma_H(2-k - \gamma_L q)))$ .

- if  $\alpha < \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$ , then the value of  $\mathcal{P1}(D)$  is zero.

$\mathcal{P2}(D)$  can be rewritten as:

$$\begin{aligned}
&\max_{\beta_H, \beta_L} q(\alpha\gamma_H + (1-\alpha)\gamma_L) \\
&\text{s. t. } E[\theta|r = g, m = h] \geq k, \\
&\quad E[\theta|r = b, m = h] \geq k, \\
&\quad 1 \geq \beta_H \geq \beta_L \geq 0.
\end{aligned}$$

The maximand is independent of  $\beta_H$  and  $\beta_L$  so we just need to ensure that the conditions are satisfied. The first condition is slack if the second condition holds (because  $E[\theta|r = g, m = h] > E[\theta|r = b, m = h]$ ) so we only need to verify that the second and third condition are feasible. Substituting for  $E[\theta|r = b, m = h]$  in the second condition and rearranging we get

$$\frac{1-\beta_H}{1-\beta_L} \geq \frac{\gamma_L(1-\alpha)k}{\gamma_H(1-k)\alpha}, \quad (14)$$

$$1 \geq \beta_H \geq \beta_L \geq 0. \quad (15)$$

We note that if (15) is satisfied, then  $\frac{1-\beta_H}{1-\beta_L} \in [0,1]$ . Therefore:

- if the RHS of (14), is bigger than one, i.e., when

$$\frac{\gamma_L(1-\alpha)k}{\gamma_H(1-k)\alpha} \geq 1 \Leftrightarrow \alpha \leq \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$$

then (14) cannot be satisfied for any  $\beta_H$  and  $\beta_L$  that satisfy (15).

- if the RHS of (14), is smaller than one, i.e., when

$$\frac{\gamma_L(1-\alpha)k}{\gamma_H(1-k)\alpha} < 1 \Leftrightarrow \alpha > \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$$

then the firm sets  $\beta_H$  and  $\beta_L$  that satisfy (14) and (15) simultaneously.

For future reference,

- if  $\alpha > \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$ , then the value of  $\mathcal{P2}(D)$  is  $q(\alpha\gamma_H + (1-\alpha)\gamma_L)$
- if  $\alpha \leq \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$  then the value of  $\mathcal{P2}(D)$  is zero.

$\mathcal{P3}(D)$  can be rewritten as:

$$\begin{aligned}
&\max_{\beta_H, \beta_L} \alpha\beta_H + (1-\alpha)\beta_L \\
&\text{s. t. } E[\theta|r = g, m = h] \geq k, \\
&\quad E[\theta|r = g, m = \emptyset] \geq k, \\
&\quad 1 \geq \beta_H \geq \beta_L \geq 0.
\end{aligned}$$

Setting the second condition binding ensures that the first condition is satisfied (because  $E[\theta|r = g, m = h] \geq E[\theta|r = g, m = \emptyset]$ ) and allows us to express  $\beta_L$ :

$$E[\theta|r = g, m = \emptyset] = k \Rightarrow \beta_L = \beta_H \frac{\alpha(1-k)(1-\gamma_H q)}{k(1-\alpha)(1-\gamma_L q)}$$

Substituting and simplifying, we can rewrite the optimization program as:

$$\begin{aligned} & \max_{\beta_H, \beta_L} \alpha \beta_H \left( 1 + \frac{(1-k)(1-\gamma_H q)}{k(1-\gamma_L q)} \right) \\ & s. t. \quad 1 \geq \beta_H \geq \beta_L \geq 0 \end{aligned}$$

Taking derivative with respect to  $\beta_H$  yields

$$\alpha \left( 1 + \frac{(1-k)(1-\gamma_H q)}{k(1-\gamma_L q)} \right) > 0$$

and therefore  $\beta_H = 1$  and  $\beta_L = \frac{\alpha(1-k)(1-\gamma_H q)}{k(1-\alpha)(1-\gamma_L q)}$  (note that  $1 \geq \beta_H \geq \beta_L \geq 0$  is satisfied because  $0 < \alpha < k < 1$  and  $0 \leq \gamma_L \leq \gamma_H \leq 1$  by assumption). For future reference, the value of  $\mathcal{P}3(D)$  is  $\alpha \left( 1 + \frac{(1-k)(1-\gamma_H q)}{k(1-\gamma_L q)} \right)$ .

$\mathcal{P}4(D)$  can be rewritten as:

$$\begin{aligned} & \max_{\beta_H, \beta_L} q(\alpha \beta_H \gamma_H + (1-\alpha)\beta_L \gamma_L) \\ & s. t. \quad E[\theta|r = g, m = h] \geq k, \\ & \quad 1 \geq \beta_H \geq \beta_L \geq 0. \end{aligned}$$

The Lagrangian is:

$$\begin{aligned} \mathcal{L}_4 = & q(\alpha \beta_H \gamma_H + (1-\alpha)\beta_L \gamma_L) \\ & + \mu_1(E[\theta|r = g, m = h] - k) + \mu_2(1 - \beta_H) + \mu_3(\beta_H - \beta_L) + \mu_4 \beta_L. \end{aligned}$$

The Karush-Kuhn-Tucker stationarity conditions are

$$\frac{\partial \mathcal{L}_4}{\partial \beta_H} = q\alpha\gamma_H + \mu_1 \frac{\partial E[\theta|r=g, m=h]}{\partial \beta_H} - \mu_2 + \mu_3 = 0 \quad (16)$$

$$\frac{\partial \mathcal{L}_4}{\partial \beta_L} = q(1-\alpha)\gamma_L + \mu_1 \frac{\partial E[\theta|r=g, m=h]}{\partial \beta_L} - \mu_3 + \mu_4 = 0, \quad (17)$$

the Karush-Kuhn-Tucker feasibility conditions are:

$$E[\theta|r = g, m = h] - k \geq 0 \quad (18)$$

$$1 - \beta_H \geq 0 \quad (19)$$

$$\beta_H - \beta_L \geq 0 \quad (20)$$

$$\beta_L \geq 0, \quad (21)$$

$$\mu_i \geq 0, \quad i = 1, 2, 3, 4. \quad (22)$$

and the Karush-Kuhn-Tucker complementarity slackness conditions are:

$$\mu_1(E[\theta|r = g, m = h] - k) = 0 \quad (23)$$

$$\mu_2(1 - \beta_H) = 0 \quad (24)$$

$$\mu_3(\beta_H - \beta_L) = 0 \quad (25)$$

$$\mu_4 \beta_L = 0. \quad (26)$$

With four complementarity slackness conditions there are  $2^4 = 16$  cases. We can immediately rule out:

- All cases with  $\mu_3 > 0, \mu_4 > 0$  (so  $\beta_H = \beta_L = 0$ ), because then  $E[\theta|r = g, m = h] = 0 < k$ , which is a contradiction;
- All cases with  $\mu_1 > 0, \mu_4 > 0$  (so  $E[\theta|r = g, m = h] = k, \beta_L = 0$ ) because if  $\beta_L = 0$ , then  $E[\theta|r = g, m = h] = 1 > k$  which is a contradiction;
- All cases with  $\mu_2 = 0$  (so  $1 > \beta_H$ ), because then  $\frac{\partial \mathcal{L}_4}{\partial \beta_H} = q\alpha\gamma_H + \mu_1 \frac{\partial E[\theta|r=g, m=h]}{\partial \beta_H} + \mu_3 \geq \mu_1 \frac{\partial E[\theta|r=g, m=h]}{\partial \beta_H} \geq 0$ , implying  $\beta_H = 1$  which is a contradiction;
- All cases with  $\mu_1 = 0$  and  $\mu_3 = 0$  (so  $E[\theta|r = g, m = h] > k$  and  $1 \geq \beta_H > \beta_L$ ) because  $\frac{\partial \mathcal{L}_4}{\partial \beta_L} = q(1-\alpha)\gamma_L + \mu_4 \geq 0$ , implying  $\beta_L = 1$  which is a contradiction;



We are left with only three cases to consider:

- $\mu_1 > 0, \mu_2 > 0, \mu_3 = 0, \mu_4 = 0, E[\theta|r = g, m = h] = k, 1 = \beta_H > \beta_L > 0$

$$E[\theta|r = g, m = h] = k \Rightarrow \beta_L = \beta_H \frac{\alpha(1-k)\gamma_H}{k(1-\alpha)\gamma_L}$$

Substituting  $\beta_H = 1$  and  $\beta_L = \frac{\alpha(1-k)\gamma_H}{k(1-\alpha)\gamma_L}$  into (17) and solving yields  $\mu_1 = \frac{\alpha\gamma_H q}{k^2} > 0$ .

Substituting  $\beta_H, \beta_L$  and  $\mu_1$  into (17) yields  $\mu_2 = \frac{\alpha\gamma_H q}{k} > 0$ . This case is feasible only if  $\beta_L = \frac{\alpha(1-k)\gamma_H}{k(1-\alpha)\gamma_L} < \beta_H = 1$ , which is equivalent to the requirement  $\alpha < \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$ .

- $\mu_1 > 0, \mu_2 > 0, \mu_3 > 0, \mu_4 = 0, E[\theta|r = g, m = h] = k, 1 = \beta_H = \beta_L > 0$

If  $1 = \beta_H = \beta_L$ , then  $E[\theta|r = g, m = h] = \frac{\alpha\gamma_H}{\alpha\gamma_H + (1-\alpha)\gamma_H}$ , i.e., the investors rationally ignore the report because it is uninformative. By  $E[\theta|r = g, m = h] = k$  it follows that this case can only be feasible when

$$\frac{\gamma_L}{\gamma_H} = \frac{\alpha(1-k)}{k(1-\alpha)} \quad (27)$$

Substituting  $1 = \beta_H = \beta_L$  and (27) into (17) yields  $\mu_1 = \frac{\alpha\gamma_H q(1-k) - k\mu_3}{(1-k)k^2}$ . Substituting  $1 = \beta_H = \beta_L$ , (27) and  $\mu_1$  into (16) yields  $\mu_2 = \frac{\alpha\gamma_H q}{k} > 0$ . Substituting  $1 = \beta_H = \beta_L, \mu_1$  and  $\mu_2$  into (16) yields  $\mu_3 = \frac{\alpha\gamma_H(1-k)q}{k} > 0$ . Then,  $\mu_1 = 0$  which is a contradiction.

- $\mu_1 = 0, \mu_2 > 0, \mu_3 > 0, \mu_4 = 0, E[\theta|r = g, m = h] > k, 1 = \beta_H = \beta_L > 0$

Substituting  $\beta_H = \beta_L = 1$  into (17) implies that  $\mu_3 = q(1-\alpha)\gamma_L > 0$ . Substituting  $\beta_H = \beta_L = 1$  and  $\mu_3$  into (16) implies  $\mu_2 = q(\alpha\gamma_H + (1-\alpha)\gamma_L) > 0$ . This case is feasible if  $E[\theta|r = g, m = h] = \frac{\alpha\beta_H\gamma_H}{\alpha\beta_H\gamma_H + (1-\alpha)\beta_L\gamma_H} = \frac{\alpha\gamma_H}{\alpha\gamma_H + (1-\alpha)\gamma_H} > k$  which is equivalent to the requirement  $\alpha > \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$ . However, we note that if  $\beta_H = \beta_L = 1$  the investors rationally

ignore the report because it is uninformative (this case is considered under a separate optimization program). Hence, this solution is not feasible.

For future reference,

- if  $\alpha > \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$ , the value of  $\mathcal{P}4(D)$  is zero.
- if  $\alpha < \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$ , then  $1 = \beta_H > \beta_L = \frac{\alpha(1-k)\gamma_H}{k(1-\alpha)\gamma_L} > 0$  and the value of  $\mathcal{P}4(D)$  is  $\frac{q\alpha\gamma_H}{k}$ .

Below is a summary of the values of the programs:

- If  $\alpha < \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$ , then
  - The value of  $\mathcal{P}1(D)$  is zero;
  - The value of  $\mathcal{P}2(D)$  is zero;
  - The value of  $\mathcal{P}3(D)$  is  $\alpha \left( 1 + \frac{(1-k)(1-\gamma_H q)}{k(1-\gamma_L q)} \right)$ ;
  - The value of  $\mathcal{P}4(D)$  is  $\frac{(1-p)\alpha\gamma_H}{k}$ .
- If  $\alpha > \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$ , then
  - The value of  $\mathcal{P}1(D)$  is:  $\frac{M_A \cdot M_B + M_C}{k(1-k)(\gamma_H - \gamma_L)}$

where  $M_A \equiv \alpha(1 - (\gamma_H(1 - k) + \gamma_L k)q)$  ,  $M_B \equiv \gamma_L k + \gamma_H(1 - k) - \gamma_H \gamma_L q$  and  $M_C \equiv \gamma_L k(1 - q(\gamma_L k + \gamma_H(2 - k) - \gamma_H \gamma_L q))$ ;

- The value of  $\mathcal{P}2(D)$  is  $q(\alpha\gamma_H + (1 - \alpha)\gamma_L)$ ;
- The value of  $\mathcal{P}3(D)$  is  $\alpha\left(1 + \frac{(1-k)(1-\gamma_H q)}{k(1-\gamma_L q)}\right)$ ;
- The value of  $\mathcal{P}4(D)$  is zero.

As a last step we compare the values of the programs. The comparison reveals that if either  $\gamma_H < \bar{g}$ , then  $\mathcal{P}3(D)$  has the highest value (for any  $\alpha \in (0,1)$ ). Hence, if Condition 1 is not satisfied the firm sets  $\beta_H^* = 1$  and  $\beta_L^* = \frac{\alpha(1-k)(1-\gamma_H q)}{k(1-\alpha)(1-\gamma_L q)}$  and the threshold is met or exceeded whenever the public report is favorable. Note that  $\beta_H^* = 1 > \beta_L^* > 0$ .

### Proof of Proposition 2 :

*Item (i):* Using the proof of Proposition 1, we note that if  $\alpha < \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$  and  $\gamma_H > \bar{g}$ , then  $\mathcal{P}4(D)$  has the highest value. The firm sets  $\beta_H^{**} = 1$  and  $\beta_L^{**} = \frac{\alpha(1-k)\gamma_H}{k(1-\alpha)\gamma_L} \in (0,1)$  and the threshold  $k$  is met whenever both the public report and the disclosure are favorable. As a last step we verify that this case is feasible, i.e., that  $\bar{g} < 1$ . This is true when  $q > \bar{q} \equiv \frac{2-k(1-\gamma_L) - \sqrt{(1-\gamma_L)(4-k(4-k(1-\gamma_L))}}{2\gamma_L}$  (note that  $\bar{q} < 1$ ).

*Item (ii):* Using the proof of Proposition 1, we note that if  $\alpha > \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$  and  $\gamma_H > \bar{g}$ , then  $\mathcal{P}1(D)$  has the highest value.

The firm sets

$$\begin{aligned} 1 &> \frac{(\gamma_H \alpha(1-k) - \gamma_L k(1-\alpha))(1-q\gamma_L)}{\alpha(\gamma_H - \gamma_L)(1-k)} \\ &= \beta_H \\ &> \beta_L \\ &= \frac{(\gamma_H \alpha(1-k) - \gamma_L k(1-\alpha))(1-q\gamma_H)}{k(\gamma_H - \gamma_L)(1-\alpha)} \\ &> 0 \end{aligned}$$

and the threshold is met or exceeded whenever either the public report or the disclosure are favorable. Using the proof of item (i), we note that this case is feasible.

**Proof of Footnote 8 claim:** The variance of conditional expectations (VCE), conditioning on the report,  $r$ , is defined as a function of the  $\beta$  vector as  $VCE(\beta) = Var[E[\theta|r]] = E[(E[\theta|r] - E[E[\theta|r]])^2]$ , which is equal to  $E[(Pr[\theta = 1|r] - \alpha)^2]$  and can be expressed as

$$VCE(\beta) = \frac{(\alpha - 1)^2 \alpha^2 (\beta_H - \beta_L)^2}{(1 - \alpha\beta_H - (1 - \alpha)\beta_L) (\alpha\beta_H + (1 - \alpha)\beta_L)}$$

Plugging in the values for  $\beta^*$  and  $\beta^{***}$  yields

$$\begin{aligned} VCE(\beta^*) &= \frac{\alpha(\alpha + k(\alpha\gamma_H q + \gamma_L(1-\alpha)q - 1) - \alpha\gamma_H q)}{q(\gamma_H(1-k) + \gamma_L k) - 1}, \text{ and} \\ VCE(\beta^{***}) &= \frac{\alpha\gamma_H(1-k) - k\gamma_L(1-\alpha)}{\alpha(\gamma_H(1-k) + \gamma_L k)} * VCE(\beta^*). \end{aligned}$$

For feasible values of the exogenous parameters, i.e.,  $0 < \alpha < k < 1$  and  $0 < \gamma_L \leq \gamma_H < 1$ , we have  $VCE(\beta^*) > VCE(\beta^{***})$ . If  $\gamma_L = 0$ , then  $VCE(\beta^*) = VCE(\beta^{***})$ .

**Proof of Proposition 3:**

$\mathcal{P1}(M)$  is identical to  $\mathcal{P4}(D)$  from the proof of Proposition 1.

$\mathcal{P2}(M)$  can be rewritten as:

$$\begin{aligned} & \max_{\beta} q(\alpha\beta_H\gamma_H + (1-\alpha)\beta_L\gamma_L) + (1-q)(\alpha\beta_H + (1-\alpha)\beta_L) \\ \text{s. t. } & E[\theta|r = g, m = h] \geq k, \\ & E[\theta|r = g, m = \emptyset] \geq k, \\ & 1 \geq \beta_H \geq \beta_L \geq 0; \end{aligned}$$

The first constraint is slack if the second constraint holds. The second constraint binds:

$$\frac{\beta_H\alpha(1-k)}{k(1-\alpha)} = \beta_L$$

and so the optimization program can be rewritten as

$$\max_{\beta} \beta_H \left( q \left( \alpha\gamma_H + (1-\alpha) \frac{\alpha(1-k)}{k(1-\alpha)} \gamma_L \right) + (1-q) \left( \alpha + (1-\alpha) \frac{\alpha(1-k)}{k(1-\alpha)} \right) \right)$$

We note that the expected payoff is increasing in  $\beta_H$  and therefore  $\beta_H = 1$ . Substituting, we find that  $\beta_L = \frac{\alpha(1-k)}{(1-\alpha)k} < 1$ . For future reference the value of the optimization program is  $\frac{\alpha}{k}(q(k\gamma_H + (1-k)\gamma_L) + (1-q))$ .

$\mathcal{P3}(M)$  is identical to  $\mathcal{P2}(D)$  from the proof of Proposition 1.

$\mathcal{P4}(M)$  can be rewritten as:

$$\begin{aligned} & \max_{\beta} (\beta_L(1-\alpha) + \alpha\beta_H) \\ \text{s. t. } & E[\theta|r = g, m = h] \geq k, \\ & E[\theta|r = b, m = h] \geq k, \\ & E[\theta|r = g, m = \emptyset] \geq k, \\ & 1 \geq \beta_H \geq \beta_L \geq 0; \end{aligned}$$

The first constraint is slack if the third constraint holds. The Lagrangean is:

$$\begin{aligned} \mathcal{L}_4 = & (1-q)(\beta_L(1-\alpha) + \alpha\beta_H) + \mu_1(E[\theta|r = b, m = h] - k) \\ & + \mu_2(E[\theta|r = g, m = \emptyset] - k) + \mu_3(1 - \beta_H) + \mu_4(\beta_H - \beta_L) + \mu_5(\beta_L) \end{aligned}$$

The Karush-Kuhn-Tucker stationarity conditions are

$$\begin{aligned} \frac{\partial \mathcal{L}_4}{\partial \beta_H} &= (1-q)\alpha + \mu_1 \frac{\partial E[\theta|r=b, m=h]}{\partial \beta_H} + \mu_2 \frac{\partial E[\theta|r=g, m=\emptyset]}{\partial \beta_H} - \mu_3 + \mu_4 = 0 \\ \frac{\partial \mathcal{L}_4}{\partial \beta_L} &= (1-q)(1-\alpha) + \mu_1 \frac{\partial E[\theta|r=b, m=h]}{\partial \beta_L} + \mu_2 \frac{\partial E[\theta|r=g, m=\emptyset]}{\partial \beta_L} - \mu_4 + \mu_5 = 0, \end{aligned}$$

the Karush-Kuhn-Tucker feasibility conditions are

$$\begin{aligned} & \frac{\alpha(1-\beta_H)\gamma_H}{\alpha(1-\beta_H)\gamma_H + (1-\alpha)(1-\beta_L)\gamma_L} - k \geq 0 \\ & \frac{\alpha\beta_H}{\alpha\beta_H + (1-\alpha)\beta_L} - k \geq 0 \\ & 1 - \beta_H \geq 0 \\ & \beta_H - \beta_L \geq 0 \\ & \beta_L \geq 0 \end{aligned}$$

and the Karush-Kuhn-Tucker complementarity slackness conditions are

$$\begin{aligned} & \left( \frac{\alpha(1-\beta_H)\gamma_H}{\alpha(1-\beta_H)\gamma_H + (1-\alpha)(1-\beta_L)\gamma_L} - k \right) \mu_1 = 0 \\ & \left( \frac{\alpha\beta_H}{\alpha\beta_H + (1-\alpha)\beta_L} - k \right) \mu_2 = 0 \\ & (1 - \beta_H) \mu_3 = 0 \\ & (\beta_H - \beta_L) \mu_4 = 0 \end{aligned}$$

$$\beta_L \mu_5 = 0$$

We know:

1.  $\beta_H > \beta_L$  because, otherwise, if  $\Pr(H|r = g, m = \emptyset) - k \geq 0$  then it has to be that  $\Pr(H|r = b, m = \emptyset) - k \geq 0$ . But we know that  $\Pr(H|r = b, m = \emptyset) < \alpha \Rightarrow \Pr(H|r = b, m = \emptyset) - k < \alpha - k < 0$ . It follows that  $\mu_4 = 0$ .
2.  $\beta_H < 1$ , because otherwise  $\Pr(H|r = b, m = h) = 0 \Rightarrow \Pr(H|r = b, m = h) - k < 0$ , so it follows that  $\mu_3 = 0$ .
3. Since  $\mu_4 = 0$ , it must be true that  $\mu_2 > 0$  because otherwise  $\frac{\partial \mathcal{L}_4}{\partial \beta_L} > 0$  implying  $\beta_L = 1$  which contradicts  $\beta_L < \beta_H < 1$ .
4. Since  $\mu_3 = 0$  ( $\beta_H < 1$ ), it must be true that  $\mu_1 > 0$  because otherwise  $\frac{\partial \mathcal{L}_4}{\partial \beta_H} > 0$  implying  $\beta_H = 1$  which is a contradiction.

It follows that  $\beta_H$  and  $\beta_L$  are defined by the binding constraints:

$$\beta_H = \frac{\gamma_H \alpha (1-k) - \gamma_L k (1-\alpha)}{\alpha (1-k) (\gamma_H - \gamma_L)} < 1$$

$$\beta_L = \frac{\gamma_H \alpha (1-k) - \gamma_L k (1-\alpha)}{k (1-\alpha) (\gamma_H - \gamma_L)} < \beta_H$$

If  $\gamma_H \alpha (1-k) - \gamma_L k (1-\alpha) < 0$ , then  $\beta_L = 0$  and the first constraint gives us

$$\frac{\gamma_H \alpha (1-k) - \gamma_L k (1-\alpha)}{\gamma_H \alpha (1-k)} = \beta_H$$

but because we assumed  $\gamma_H \alpha (1-k) - \gamma_L k (1-\alpha) < 0$ , this implies  $\beta_H < 0$ , which is not feasible. The second constraint gives us  $1 - k = 0$ , contradicts our assumption of  $1 > k$ . So  $\mathcal{P4}(M)$  has a solution only for  $\gamma_H > \frac{k(1-\alpha)}{\alpha(1-k)} \gamma_L$ . For future reference, the value of the optimization program is  $q(\alpha \gamma_H + \gamma_L (1-\alpha)) + (1-q) \frac{\alpha(1-k)\gamma_H - k(1-\alpha)\gamma_L}{(1-k)(\gamma_H - \gamma_L)k}$ .

$\mathcal{P5}(M)$  can be rewritten as:

$$\max_{\beta} \alpha \beta_H + (1-\alpha) \beta_L$$

$$s. t. \quad E[\theta|r = g, m = h] \geq k,$$

$$E[\theta|r = g, m = \emptyset] \geq k,$$

$$E[\theta|r = g, m = l] \geq k,$$

$$1 \geq \beta_H \geq \beta_L \geq 0$$

The first and second constraints are slack if the third constraint is satisfied. The expected payoff is increasing in both  $\beta_H$  and  $\beta_L$ . We examine the third constraint and note that:

$$\frac{\partial}{\partial \beta_H} \left( \frac{\alpha \beta_H (1-\gamma_H)}{\alpha \beta_H (1-\gamma_H) + (1-\alpha) \beta_L (1-\gamma_L)} \right) \propto (1-\gamma_L)(1-\gamma_H)(1-\alpha) \alpha \beta_L > 0$$

This suggests  $\beta_H = 1$ .  $\beta_L$  will be defined by

$$0 = \frac{\alpha (1-\gamma_H)}{\alpha (1-\gamma_H) + (1-\alpha) \beta_L (1-\gamma_L)} - k$$

$$\beta_L = \frac{\alpha (1-k)(1-\gamma_H)}{k(1-\alpha)(1-\gamma_L)}$$

For future reference the value of the optimization program is  $\alpha \left( \frac{k(1-\gamma_L) + (1-k)(1-\gamma_H)}{k(1-\gamma_L)} \right)$ .

$\mathcal{P6}(M)$  can be rewritten as:

$$\max_{\beta} \alpha \beta_H + (1-\alpha) \beta_L + q(\alpha(1-\beta_H)\gamma_H + (1-\alpha)(1-\beta_L)\gamma_L)$$

$$s. t. \quad E[\theta|r = g, m = h] \geq k,$$

$$E[\theta|r = g, m = \emptyset] \geq k,$$

$$E[\theta|r = b, m = h] \geq k,$$

$$E[\theta|r = g, m = l] \geq k, \\ 1 \geq \beta_H \geq \beta_L \geq 0.$$

The first and second constraints are slack if the third and fourth are satisfied. Hence, the Lagrangean is

$$\begin{aligned} \mathcal{L}_6 = & \alpha\beta_H + (1 - \alpha)\beta_L + q(\alpha(1 - \beta_H)\gamma_H + (1 - \alpha)(1 - \beta_L)\gamma_L) \\ & + \mu_1(E[\theta|r = b, m = h] - k) + \mu_2(E[\theta|r = g, m = l] - k) \\ & + \mu_3(1 - \beta_H) + \mu_4(\beta_H - \beta_L) + \mu_5(\beta_L) \end{aligned}$$

The Karush-Kuhn-Tucker stationarity conditions are

$$\begin{aligned} \frac{\partial \mathcal{L}_6}{\partial \beta_H} = & \alpha(1 - \gamma_H q) \\ & + \mu_1 \frac{\partial E[\theta|r=b, m=h]}{\partial \beta_H} + \mu_2 \frac{\partial E[\theta|r=g, m=l]}{\partial \beta_H} - \mu_3 + \mu_4 = 0 \\ \frac{\partial \mathcal{L}_6}{\partial \beta_L} = & (1 - \alpha)(1 - \gamma_L q) \\ & + \mu_1 \frac{\partial E[\theta|r=b, m=h]}{\partial \beta_L} + \mu_2 \frac{\partial E[\theta|r=g, m=l]}{\partial \beta_L} - \mu_4 + \mu_5 = 0, \end{aligned}$$

the Karush-Kuhn-Tucker feasibility conditions are

$$\begin{aligned} \frac{\alpha(1 - \beta_H)\gamma_H}{\alpha(1 - \beta_H)\gamma_H + (1 - \alpha)(1 - \beta_L)\gamma_L} - k & \geq 0 \\ \frac{\alpha\beta_H(1 - \gamma_H)}{\alpha\beta_H(1 - \gamma_H) + (1 - \alpha)\beta_L(1 - \gamma_L)} - k & \geq 0 \\ 1 - \beta_H & \geq 0 \\ \beta_H - \beta_L & \geq 0 \\ \beta_L & \geq 0 \end{aligned}$$

and the Karush-Kuhn-Tucker complementarity slackness conditions are

$$\begin{aligned} \mu_1(E[\theta|r = b, m = h] - k) & = 0 \\ \mu_2(E[\theta|r = g, m = l] - k) & = 0 \\ \mu_3(1 - \beta_H) & = 0 \\ \mu_4(\beta_H - \beta_L) & = 0 \\ \mu_5(\beta_L) & = 0 \end{aligned}$$

We know:

1.  $\mu_4 = 0$  (and so  $\beta_H > \beta_L$ ) because otherwise, if  $\Pr(H|r = g, m = \emptyset) - k \geq 0$  then it has to be that  $\Pr(H|r = b, m = \emptyset) - k \geq 0$ . But we know that  $\Pr(H|r = b, m = \emptyset) < \alpha \Rightarrow \Pr(H|r = b, m = \emptyset) - k < \alpha - k < 0$  which is a contradiction.
2.  $\mu_3 = 0$  (and so  $\beta_H < 1$ ), because otherwise  $\Pr(H|r = b, m = h) = 0$  which implies  $\Pr(H|r = b, m = h) - k < 0$  (and contradicts the constraint).
3. Since  $\mu_3 = 0$  ( $\beta_H < 1$ ), then it must be true that  $\mu_1 > 0$  because otherwise  $\frac{\partial \mathcal{L}_6}{\partial \beta_H} > 0$  implying  $\beta_H = 1$  which is a contradiction.
4. Since  $\mu_4 = 0$ , then it must be true that  $\mu_2 > 0$  because otherwise  $\frac{\partial \mathcal{L}_6}{\partial \beta_L} > 0$  implying  $\beta_L = 1$  which contradicts  $\beta_L < \beta_H < 1$ .

We note that  $\beta_H$  and  $\beta_L$  are defined by the first and second constraints binding:

$$\beta_H = \frac{(\gamma_H \alpha(1 - k) - \gamma_L k(1 - \alpha))(1 - \gamma_L)}{(\gamma_H - \gamma_L) \alpha(1 - k)}$$

$$\beta_L = \frac{(\gamma_H \alpha(1-k) - \gamma_L k(1-\alpha)) (1-\gamma_H)}{(\gamma_H - \gamma_L) k(1-\alpha)}$$

We need  $(\gamma_H \alpha(1-k) - \gamma_L k(1-\alpha)) > 0$  for  $\beta_H$  and  $\beta_L$  to be non-negative. If this condition does not hold then  $\beta_L = 0$  and

$$\begin{aligned} \frac{\alpha(1-\beta_H)\gamma_H}{\alpha(1-\beta_H)\gamma_H + (1-\alpha)\gamma_L} - k &= 0 \\ \frac{\alpha(1-k)\gamma_H - k(1-\alpha)\gamma_L}{\alpha(1-k)\gamma_H} &= \beta_H \end{aligned}$$

but because we assumed  $\gamma_H \alpha(1-k) - \gamma_L k(1-\alpha) < 0$ , this implies  $\beta_H < 0$ , which is not feasible. So  $\mathcal{P6}(M)$  has a solution only for  $\gamma_H > \frac{k(1-\alpha)}{\alpha(1-k)}\gamma_L$ . Lastly, we note that  $\beta_H < 1$  because

$$\begin{aligned} 1 - \beta_H &= 1 - \frac{(\gamma_H \alpha(1-k) - \gamma_L k(1-\alpha)) (1-\gamma_L)}{(\gamma_H - \gamma_L) \alpha(1-k)} \\ &\quad \alpha(1-k)(\gamma_H - \gamma_L) - (1-\gamma_L)(\gamma_H \alpha(1-k) - \gamma_L k(1-\alpha)) \\ &= \gamma_L(\gamma_H \alpha(1-k) - \gamma_L k(1-\alpha)) + \gamma_L(k - \alpha) \\ &> 0, \end{aligned}$$

by assumption. For future reference, the value of the optimization program is:

$$M_D \cdot (k(1-\gamma_L) + (1-k)(1-\gamma_H) + \gamma_H \gamma_L q) + (k-\alpha)\gamma_H \gamma_L q$$

where  $M_D \equiv \frac{(\gamma_H \alpha(1-k) - \gamma_L k(1-\alpha))}{(\gamma_H - \gamma_L)(1-k)k}$ .

Below is a summary of the values of the programs:

- If  $\alpha < \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$ , then
  - The value of  $\mathcal{P1}(M)$  is  $\frac{q\alpha\gamma_H}{k}$ ;
  - The value of  $\mathcal{P2}(M)$  is  $\frac{\alpha}{k}(q(k\gamma_H + (1-k)\gamma_L) + (1-q))$ ;
  - The value of  $\mathcal{P3}(M)$  is zero;
  - The value of  $\mathcal{P4}(M)$  is zero;
  - The value of  $\mathcal{P5}(M)$  is  $\alpha \left( \frac{k(1-\gamma_L) + (1-k)(1-\gamma_H)}{k(1-\gamma_L)} \right)$ ;
  - The value of  $\mathcal{P6}(M)$  is zero;
- If  $\alpha > \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$ , then
  - The value of  $\mathcal{P1}(M)$  is zero;
  - The value of  $\mathcal{P2}(M)$  is  $\frac{\alpha}{k}(q(k\gamma_H + (1-k)\gamma_L) + (1-q))$ ;
  - The value of  $\mathcal{P3}(M)$  is  $q(\alpha\gamma_H + (1-\alpha)\gamma_L)$ ;
  - The value of  $\mathcal{P4}(M)$  is  $q(\alpha\gamma_H + \gamma_L(1-\alpha)) + (1-q) \frac{\alpha(1-k)\gamma_H - k(1-\alpha)\gamma_L}{(1-k)(\gamma_H - \gamma_L)k}$ ;
  - The value of  $\mathcal{P5}(M)$  is  $\alpha \left( \frac{k(1-\gamma_L) + (1-k)(1-\gamma_H)}{k(1-\gamma_L)} \right)$ ;
  - The value of  $\mathcal{P6}(M)$  is  $\frac{\alpha(1-\gamma_H(1-k) - \gamma_L k)(\gamma_L k + \gamma_H(1-\gamma_L q - k)) - M_D}{(\gamma_H - \gamma_L)(1-k)k}$  where  $M_D \equiv \gamma_L k(1 - \gamma_L k - \gamma_H(1 + (1 - \gamma_L)q - k + q))$ .

It is immediate that in case (B) the value of program  $\mathcal{P3}(M)$  is lower than the value of program  $\mathcal{P4}(M)$ . So we only need to consider:

- If  $\alpha < \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$ , the values of  $\mathcal{P1}(M)$ ,  $\mathcal{P2}(M)$  and  $\mathcal{P5}(M)$ .

- If  $\alpha > \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$ , the values of  $\mathcal{P}2(M)$ ,  $\mathcal{P}4(M)$ ,  $\mathcal{P}5(M)$  and  $\mathcal{P}6(M)$ .

As a last step we consider the case when Condition 1 is not satisfied and compare the values of the programs above with the value of program  $\mathcal{P}3(D)$  (the highest value program under the discretion regime as shown in Proposition 1). The comparison reveals that the value of program  $\mathcal{P}3(D)$  is strictly larger than:

- the values of programs  $\mathcal{P}1(M)$  and  $\mathcal{P}5(M)$ ;
- the value of program  $\mathcal{P}2(M)$  if  $\gamma_H < g_o$ , where  $g_o \equiv \frac{1-\gamma_L(1-\gamma_L(1-k))q}{1-\gamma_L k q} > 0$ ;
- the value of program  $\mathcal{P}6(M)$  because the value of program  $\mathcal{P}6(M)$  is lower than the value of program  $\mathcal{P}1(D)$  which is lower than the value of program  $\mathcal{P}3(D)$ ;
- the value of program  $\mathcal{P}4(M)$  because the value of program  $\mathcal{P}4(M)$  is lower than the value of program  $\mathcal{P}6(D)$  if  $\gamma_H < g_{oo}$ , where  $g_{oo} \equiv \frac{q-\gamma_L(1-(1-q)(1-k))}{1-\gamma_L q-k(1-q)}$ . Feasibility

requires that  $g_{oo} > 0$  which holds when  $p < \frac{1-\gamma_L}{1-\gamma_L(1-k)}$ . Further, (as we show above) the

value of program  $\mathcal{P}6(M)$  is lower than the value of program  $\mathcal{P}3(D)$ ;

It follows that a sufficient condition for discretion to be strictly valuable is that  $\gamma_H < \min\{\bar{g}, g_o, g_{oo}\}$  and  $p < \frac{1-\gamma_L}{1-\gamma_L(1-k)}$ .

#### **Proof of Proposition 4 :**

*Item (i):* Using the proof of Proposition 3, we consider the case when Conditions 1 and 2 are not satisfied and compare the values of programs  $\mathcal{P}1(M)$ ,  $\mathcal{P}2(M)$  and  $\mathcal{P}5(M)$  (the relevant programs when Condition 2 is not satisfied) with the value of program  $\mathcal{P}4(D)$  (the highest value program under the discretion regime as shown in Proposition 2, item (i)). The comparison reveals that the value of program  $\mathcal{P}4(D)$ :

- is equal to the value of program  $\mathcal{P}1(M)$ ;
- is strictly larger than the value of program  $\mathcal{P}2(M)$  if  $\gamma_H > \frac{\gamma_L(1-k)q+1-q}{(1-k)q} \equiv g^o$ .

Feasibility requires that  $g^o < 1$  which holds if  $q > \frac{1}{2-\gamma_L(1-k)-k}$ ;

- is strictly larger than the value of program  $\mathcal{P}5(M)$

It follows that the firm is indifferent between discretion and mandatory disclosure if  $\gamma_H > \max\{\bar{g}, g^o\}$  and  $q > \frac{1}{2-\gamma_L(1-k)-k}$ .

*Item (ii):* Using the proof of Proposition 3, we consider the case when Conditions 1 and 2 are satisfied and compare the values of programs  $\mathcal{P}2(M)$ ,  $\mathcal{P}4(M)$ ,  $\mathcal{P}5(M)$  and  $\mathcal{P}6(M)$  (the relevant programs when Condition 2 is satisfied) with the value of program  $\mathcal{P}1(D)$  (the highest value program under the discretion regime as shown in Proposition 2, item (ii)). The comparison reveals that the value of program  $\mathcal{P}1(D)$  is strictly lower than the value of program  $\mathcal{P}4(M)$  if  $\gamma_H > \frac{1-\gamma_L}{1-\gamma_L(1-p)} \equiv g^{oo}$ . We note that  $g^{oo} < 1$  (so  $\gamma_H > g^{oo}$  is feasible). It immediately follows that the firm strictly prefers mandatory disclosure if  $\gamma_H > \max\{\bar{g}, g^{oo}\}$ .

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